Modern Science

EDITED BY SIR JOHN LUBBOCK BART.

LAWS AND PROPERTIES OF MATTER

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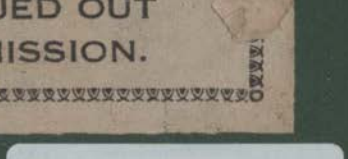

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LAWS AND PROPERTIES OF MATTER

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LAWS AND PROPERTIES

OF

MATTER

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BY

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PREFACE

THE following pages have been written as an introduction to the study of the Properties of Matter. The book does not pretend to be in any way a complete treatise. It aims at making clear to one who wishes to understand something of Physics the meaning of the terms applied to matter, and the principal properties it possesses. The subject cannot be fully treated without mathematical reasoning, though in the book itself only a little elementary algebra is used occasionally. I hope that most of the contents may be intelligible and interesting even to those who do not follow the mathematical reasoning when it occurs. In conclusion, my thanks are due to Mr. L. R. WILBERFORCE, for his kindness and assistance in reading the proofs.

R. T. GLAZEBROOK.

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LAWS AND PROPERTIES

OF

MATTER

CHAPTER I

INTRODUCTORY—UNITS OF MEASUREMENT

In this book we propose to deal in an elementary manner with some of the simpler laws and properties of matter. We shall have to treat of the various forms in which matter is known to us, and of the actions which go on between them. Most of the properties which will come before us are capable of exact definition and measurement, and a brief notice is required of the fundamental units in terms of which those measurements are made.

We commence, then, in this introductory chapter, with a few explanations and definitions of names and terms which will frequently occur.

Measurement of Space.—Space, like any other physical quantity, must be measured in terms of a unit

of its own kind. When we say that a certain room is 20 feet long, we imply that we have taken a certain length to which the name *foot* is given, and find that twenty of these placed end to end make up the length of the room.

The unit of length usually adopted now for scientific purposes is the centimetre, which is the one-hundredth part of a metre. A metre was defined, by a law of the French Republic in 1795, as the distance between the ends of a rod of platinum made by Borda, the temperature of this rod being that of melting ice. At that date the distance, along an arc of the meridian of the earth's surface, between the pole and the equator had recently been measured, and it was thought that Borda's platinum rod represented one ten-millionth part of this. Further research has shown that this is not strictly the case, so that the standard of length is not the terrestrial globe, but Borda's platinum rod. The divisions of the metre are decimal. Thus,

10 decimetres = 1 metre
10 centimetres = 1 decimetre
10 millimetres = 1 centimetre

Measurement of Mass.—The quantity of matter in a body is called its Mass.

The mass of any body is measured in terms of some unit of mass. A mass 10 means one which contains ten times as much matter as the unit, whatever it be. The unit of mass on the metric system is the kilogramme,

which is the quantity of matter in a lump of platinum made by Borda and intended to represent the mass of a cubic decimetre of distilled water at a temperature of 4° C. The unit of mass we shall adopt is the gramme, which is one thousandth part of Borda's standard, the kilogramme. A gramme is approximately the mass of one cubic centimetre of water at 4° C., and this fact leads to great simplification in many approximate numerical calculations, but it must be remembered that the definition of the gramme is that it contains one thousandth part of the matter contained in Borda's standard of platinum, not that it is the mass of 1 c.c. of water.

Measurement of Time.—Another fundamental physical quantity with which we have to deal is Time. The idea of time (says Maxwell) in its most primitive form is probably the recognition of an order of sequence in our states of consciousness. The measure of time is derived from the apparent motion of the stars. The time occupied by the rotation of the earth round its axis is very nearly indeed constant, but owing to the motion of the earth round the sun the interval between two successive passages of the sun across the meridian of any place differs from day to day. The average of such intervals during a year is a mean solar day. A mean solar day contains 86,400 seconds, and the fundamental unit of time is the mean solar second.

We shall find, as we proceed, that other physical quantities can be expressed in terms of these three units

of space, mass, and time. Each physical quantity is measured in terms of a unit of its own kind, and the measurement of a quantity in terms of the fundamental units resolves itself into two parts. We require to know (1) the number of times the unit is contained in the quantity in question, and (2) the relation between this unit and the fundamental units of mass, length, and time. A quantity so determined is measured in absolute measure. When we take the centimetre, gramme, and second as fundamental units we are said to be using the C.G.S. system. This is now generally adopted for scientific purposes.

Measurement of Position.—The position of one point relative to another, or the relative position of two points, is known when the length and direction of the line joining them is known.

Change of Position—Motion.—When the line joining two points varies either in length or direction, then either point is in motion relatively to the other. Motion is measured by change in position.

Rate of Change of a Quantity.—In many problems we have to deal with the change which takes place in a quantity, such as the position of a body relative to another, in a given time. This is measured by the rate of change of the quantity. The rate of change, when uniform, is measured by the change which takes place in the unit of time; when variable, it is measured by the ratio of the change taking place in a given interval to the interval, when the interval is made so

short that during it the change may be considered as occurring uniformly.

Velocity.—The velocity of a point is the rate of change in its position. When uniform, it is measured by the change in position in a unit of time, when variable, by the ratio of the change occurring in a very small interval to that interval.

To determine completely the measure of a velocity we require to know both the distance the body moves in a given interval and the direction in which it moves.

Speed.—The rate at which a body moves, as measured when uniform by the distance traversed per unit time, or by the ratio of the distance traversed in a very short interval to that interval, is called the speed of the body. Thus the measure of speed does not involve that of the direction of motion, but only of the length moved over.

Acceleration.—When the velocity of a body is not uniform, it is said to have acceleration. This is measured by the rate of change of the velocity—that is, by the change in velocity per second; and since velocity is measured by space traversed per second, acceleration is measured by space per second per second.

If we are given the acceleration of a particle and know its position and velocity at one instant, it becomes a mathematical problem to find the position and velocity at any future time.

In some cases, *e.g.* when the acceleration is constant in direction and magnitude, the reasoning is simple, though it is beyond our limits to enter into it here; in others again the difficulties of the problem exceed our present mathematical powers.

CHAPTER II

MOTION—KINETICS—THE RELATION BETWEEN FORCE AND MOTION—LAWS OF MOTION

So far we have been dealing with geometrical relations connected with the position and motion of a particle, without inquiring into the cause of the motion or into the connection between the cause of the motion and the amount of motion produced. This first part of our subject is called kinematics. We have now to consider kinetics, which treats of the relation between force and motion.

We commence with the following definition: That which changes or tends to change the state of rest or motion of a body is called force. Our primary idea of force seems to come from our sense of resistance opposed to muscular effort on our part. If we try to lift a weight we experience a sense of resistance to our endeavours, and it becomes necessary to exert force. The result of this force may be to lift the weight, thus producing motion, or the force we can exert may not be sufficient to overcome the other forces acting on the

weight, and we may only tend to change the state of rest of the weight.

Again, force, when applied to a body, in addition to producing motion of the body as a whole, has another effect—it changes, to a greater or less degree, the shape and possibly the size of the body, producing relative motion of its parts, even when no motion of the body as a whole ensues. The amount of this deformation depends, of course, on the nature of the material of the body. A great force is required to change the shape of a mass of iron or rock, while the force necessary to produce the same change in a similar mass of jelly would be extremely small.

Rigidity is the name given to the property of matter which enables it in certain forms to resist change of shape, while the change of volume produced in a body by a force depends on the resistance it can offer to compression. A body, the form or volume of which is changed, owing to the action of force, is said to be strained.

These two effects of force lead to two ways of measuring forces. We may measure them either (1) by the motion they can produce in a certain body; or (2) by the strains they cause in some definite body. In a spring balance we have an example of this last method of measuring force. When a force is applied to a body and strains it, the strain, of course, in general is not confined to the immediate neighbourhood of the point at which the force is applied. Owing to the strain, forces

tending to resist further strain are called into play throughout the body. The name stress is usually given to such forces. In the case of the spring balance the spring is extended, that is, the body is strained until the stresses called into play balance the impressed forces.

To understand how to obtain a measure of a force from the amount of motion it can produce in a given mass, we must consider carefully Newton's laws of motion, and endeavour to give some account of the reasoning on which they are based.

The laws are three in number. We may state the first law thus:

Law I.—Every body continues in a state of rest or of uniform motion in a straight line unless compelled to change that state by the action of some external force.—The law expresses the principle of inertia in matter. A lump of dead matter at rest will not move of itself; an external agency of some sort is required to start it. A body in motion, freed from all external forces, will not come to rest of itself. It requires the action of some external agency. The truth of the first part of the statement needs no experiment or argument to establish it; the second part is not so obvious. We cannot get a body on which to experiment which shall be free from all external force. All we can do by direct experiment is to show that the smaller the resistance to the motion the farther will the body move. Thus, if we slide a stone along a smooth, level surface, the

only force which opposes the motion is the friction between the surface and the stone, and the smoother we make the surface, the further the stone will travel. On ice it would go a much longer distance than on a rough road. Or again, a properly supported pendulum will continue to vibrate for a long time till brought to rest by the resistance of the air and the friction at the support. If we enclose it in a chamber from which all the air is exhausted, one of the forces which previously retarded its motion is removed and the motion will continue much longer. But while such observations as these tend to make us believe that the law may be true, they do not establish it. For this we have recourse to reasoning of the following character; we endeavour to solve some complicated problem in astronomy or mechanics on the assumption that this and the other laws of motion are true, and we find that in all cases the observed facts tally with our predictions founded on the above assumptions. We can foretell eclipses of the sun and determine over what portion of the earth they will be visible. We can solve a number of astronomical questions, and our results accord with our observations. We therefore infer that the fundamental premises of our argument, the laws of motion, are sound. We adopt this method continually in scientific work, and reasoning of this kind forms the basis of most of our scientific beliefs.

In our discussion of the second law, a technical term which will occur requires some explanation.

Every one knows that the effort required to stop a moving body depends to some extent upon the mass of the body. A cannon ball, even though moving slowly, is stopped much less easily than a cricket ball; a blow that would drive a small stone many yards will hardly stir a lump of rock. Again, the effort also depends on the velocity of the body, much more force is required to throw a ball fifty yards than to toss it a few feet; and so we find that the product of the mass and the velocity comes into our considerations. This product is called momentum. In some of the older writings on the subject the term 'quantity of motion' is used for momentum, but the phrase is not a good one. The momentum of a body of mass m grammes moving with a velocity of v centimetres per second is mv.

Now, force has been defined as that which changes or tends to change the state of rest or motion of a body, but as yet we have had no statement as to how force is measured. It may reasonably be measured by the change it produces in the motion of the body; but since the effect of a given force will depend on the mass of the body to which it is applied, we cannot fairly estimate the force by the change in velocity, but must consider the mass as well, and interpret change of motion as being equivalent to change of momentum. But again, the mechanical effect of any muscular effort will depend on the time for which that effort is sustained. If we apply a force to a body for a long time, the momentum generated will be greater than it would be if the appli-

cation of the force were instantaneous; and so, if we are to measure force by change of momentum, it must be by change in some definite time. We take, then, as a measure of force the change of momentum it produces in a unit of time and thus arrive at the following statement.

Law II.—Rate of change of momentum is proportional to the impressed force, and takes place in the direction in which the force acts.

If we proceed as above, we may look upon this second law as a dynamical definition and measure of force. When the momentum of a body is being changed, it must be by the action of some external agency. We take the rate of change of momentum as a measure of this agency, and call it the force. Still we must be consistent with ourselves, and ask, Will the two methods of measuring forces we have described—the strain method and the momentum method—lead to the same result? A certain force produces a certain definite amount of momentum per second, and a certain definite effect in straining a body. Will another force, which, when measured by its straining effect, is twice as great as the former, produce per second twice the momentum? For a reply to this we must have recourse to experiment, and many experiments might be devised to test it. In each of these the answer is found to be affirmative; the dynamical measure, the rate of change of momentum, is proportional to the statical measure of the force.

Now, in all cases in which the mass of the body remains unchanged, rate of change of momentum is measured by the mass multiplied by the rate of change of velocity, and this last quantity we have called acceleration. Thus, the force producing acceleration a in a mass m is proportional to the product $m\,a$. If we choose our unit of force properly, we may say that the force is equal to the product of the mass and the acceleration. We can do this by defining as the unit of force that force which will produce unit acceleration in unit of mass—that is to say, which generates a unit of momentum per second. A force containing F of such units will generate per second F units of momentum, and we have, therefore, $F = m\,a$.

If we take the centimetre and gramme as units, this unit force is called a dyne.

Thus a dyne is a force which generates per second a velocity of 1 cm. per second in a mass of 1 gramme. The English unit of force is called a poundal. A poundal is a force which generates per second a velocity of 1 foot per second in a mass of 1 lb.

The consideration of the third law of motion in its fullest meaning will occupy us at some length in a future chapter. It may be stated thus:

Law III.—To every action there is always an equal and opposite reaction.

Limiting for the present the term action to the action of a force, the law tells us that if a body, A, acts on a second body, B, with a certain force, B reacts on A

with an equal force. Thus, when a weight is resting on a table, the upward action of the table in supporting the weight is equal and opposite to the downward force with which the weight presses on the table. Or again, when a bullet is fired from a gun, the action of the gun, through the powder on the bullet, measured by the momentum produced in the bullet, is equal to the action of the bullet on the gun. This last is shown in the momentum of the recoil, generated in the same time as that of the bullet, and the third law states that these two momenta are equal. Or again, when one billiard-ball strikes a second, it loses some of its momentum. This loss of momentum measures the impulse it exerts on the second, and is shown in an equal gain of momentum in that ball, acquired in the same interval of time. On the whole, taking the two balls, there is neither loss nor gain of momentum, and this is in accordance with the second law, for no external or impressed force has acted on the two balls together.

But, as we shall see shortly, the third law is capable of a wider signification than this.

We have spoken several times of the weight of a body; let us analyse a little closer the meaning of this term. We know that if we hang a body up by a string the string is stretched, and, if we cut the string, the body falls towards the ground. There is clearly a force acting on it, pulling it in a vertical direction. To this force we give the name weight. This weight is the

resultant of an immense number of attractions taking place between the particles of the body and the particles of the earth itself. The law governing these individual attractions was discovered by Newton, and is known as the law of gravitation. It may be stated as follows:

Law of Gravitation.—There is an attraction between any two particles of matter in the universe. The amount of this attraction is directly proportional to the product of the masses of the particles, and inversely proportional to the square of the distance between them.

So that, if we have two particles of mass m, m' respectively, at a distance a apart, the force between them is proportional to $\frac{m\,m'}{a^2}$. Taking, then, the case of the earth and any other body, the resultant of all the attractions between the innumerable particles of the two is called the weight of the body. For a body whose size is small, compared with that of the earth, this resultant is a force which acts through a definite point in the body, and this point is called the centre of gravity of the body. The weight of a body is thus a force which is properly measured in dynes or poundals as the case may be.

Now, the weight of a body causes it to fall to the earth when it is set free, and the velocity with which it falls continually increases; it has acceleration. Let us denote this acceleration by g, let w dynes be the weight of the body, and m grammes its mass. Then,

since the force producing motion is measured by the product of the mass of the body and the acceleration produced, we have the relation $w = m g$.

Now, experiments of various kinds show that, at a given point on the earth, the acceleration of all falling bodies is the same; so that g is a constant for all bodies, and the weight of a body is proportional to its mass. We can readily see that, assuming Newton's law of attraction, this result must follow. We may look upon the experimental result that g is constant as a verification of this part of the law. The simplest experiment on the point is due to Galileo, who allowed balls of different masses to fall from the top of the leaning tower in Pisa, and found that they all reached the ground in the same time. If the shapes of the bodies be very different, so that they expose different amounts of surface to the action of the air, this will not be the case, because of the resistance offered by the air, which depends on the surface and not on the mass of the body. This difficulty is met by performing the experiment in a vacuum, and so removing the effect due to the air, when it will be found that two bodies of very different mass, such as a sovereign and a feather, will fall at the same rate. The same fact is proved by experiments with a simple pendulum. A simple pendulum consists of a heavy mass suspended from a fixed point by a fine string, so fine that we may omit from consideration its weight when compared with that of the mass. Such a system when

displaced will oscillate backwards and forwards, and it is found that the oscillations are isochronous, that is, that each occupies the same interval of time. Moreover, it is found that this time, at a given point of the earth's surface, depends only on the length of the pendulum, and not at all upon the mass of the bob. If we have two pendulums with bobs of the same size but of different mass, the one of lead, say, and the other of wood, suspended side by side by strings of the same length, and start them simultaneously, it will be found that they will keep time together for a considerable period. The pendulum with the wooden bob will come to rest before the other, because the air resistance will have the greater effect on it, and this air resistance will also slightly affect the period, but only to a secondary extent. The time of oscillation will be the same for the two, because though the mass of the lead bob is the greater, yet so also is the weight of this bob, and that in the same proportion. Thus the acceleration produced in the lead bob is the same as that produced in the wooden one.

We can show by various experiments that the value of g varies from point to point of the earth's surface. Thus, a pendulum which makes one oscillation per second near the equator, will complete it in less than a second when suspended near the poles; or a mass which stretches a spring balance by a certain amount near the poles will extend it less, that is, will weigh less, if the balance and body be carried to the

equator. The mass of a body is of course a constant quantity, and can only be changed by removing a part of the body. Its weight, or the force with which it is pulled to the centre of the earth, depends on its position, and changes as that position changes. Near the pole the body is nearer the earth's centre, and appears therefore to weigh more than at the equator, and this effect is increased by the consequences of the earth's rotation round its axis.

The fact that at a given point the weight of a body is proportional to its mass, and perhaps also the want of a proper nomenclature, has led to some confusion between weight and mass. A pound or a gramme is often used somewhat loosely for the mass of a pound or gramme or for the weight or force with which the pound or gramme is attracted to the earth, and the consequent confusion is increased by the fact that in the ordinary use of a balance the masses of two bodies are compared by comparing their weights.

With such a balance we compare the force with which a pound of tea or of sugar is attracted to the earth with the force with which the iron weights are attracted. When these two forces are equal, the mass of tea is equal to the mass of the weights—omitting a small correction due to the buoyancy of the air—and it is this mass, or quantity of tea stuff, for which we pay our money, not the force with which it is attracted to the earth.

We may notice that the ordinary balance would

not enable us to prove that the weight of a body varies from point to point on the earth, for the 'weights' or standard masses with which we make our comparisons will vary also in weight to the same amount. A spring balance, on the other hand, measures force not mass, and if correctly graduated for the latitude of London, say, will not measure the same mass elsewhere. The mass required to give the same reading would be greater at the equator, and less at the pole than at London. A customer buying goods would get more than he should at the equator, less than he should at the pole.

Weight is a force, and is properly measured as such in dynes or poundals, while mass is given in grammes or pounds.

The equation $w = m g$ expresses the fact that, using C.G.S. units, the number of dynes in a given mass is g times the number of grammes in that mass, while a similar statement will hold for other systems of units. Now, experiment has shown that in the latitude of London the value of g is about 981 centimetres per second per second, so that the weight of 1 gramme at London contains about 981 dynes.

Galileo's experiment teaches us that the cceleration of two falling bodies is the same. We may infer from a similar experiment that the acceleration of a falling body is—apart from the effects of the air—a constant at all points of its fall; for we can show that if a body is moving with uniform acceleration, the space described in a given time is proportional to the square of the

time, and, by observing the spaces described by a falling body in 1, 2, 3, &c. seconds, we find that they are proportional to 1^2, 2^2, 3^2, &c. Thus the acceleration is constant. These same observations give us a value for g, for it can be proved that the space passed over in time t by a falling body is $\frac{1}{2} g t^2$. Now, it is found that in London a body falls in one second from rest about 410·5 cm., hence $g = 981$ cm. per second per second. This is not a good method of measuring g, for it is difficult to measure an interval of one second with very great accuracy, while the distance moved over in a longer time is too great to be convenient.

We may of course express g in terms of feet per second per second, for 410·5 cm. is about 16·1 feet. Thus the body falls about 16·1 feet in one second from rest, and hence g is 32·2 feet per second per second.

Various devices have been employed to obtain motion with uniform acceleration less than g, but depending upon it. In Atwood's machine a thin string hangs over a very light pulley. This is mounted on friction wheels to run very easily, and two equal masses are supported at the ends of the string. Suppose now a very small mass is placed on one of the larger masses; then this mass will begin to descend. The force producing motion, if we suppose the friction of the pulley negligible, is the weight of the small mass, while the mass moved, if we do not consider the pulley, will be the sum of the two large masses and the small mass. Let us call each of the large masses M, and the

small mass m grammes; then, if a is the acceleration, the rate of change of the momentum is $(2\,\mathrm{M} + m)a$, and this is equal to the force or $m\,g$.

Thus, $$(2\,\mathrm{M} + m)a = m\,g;$$

or, $$a = \frac{m}{2\,\mathrm{M} + m}\,g.$$

This result is clear, without writing down the equations, if we reflect that the weight of the small mass acting on itself produces an acceleration g and that the acceleration produced by a given force is inversely proportional to the mass on which it acts. So that the acceleration generated in Atwood's machine will be to g in the ratio of m to $2\,\mathrm{M} + m$, or, as above,

$$a = \frac{m}{2\,\mathrm{M} + m}\,g.$$

Thus, suppose we take each of the masses M to be 495 grammes, and m to be 10 grammes; then we have $2\,\mathrm{M} + m = 1000$ grammes, and $a = \frac{1}{100} \cdot g$. Now, a body moving with this acceleration would describe, we should find, spaces of about 4·9, 19·6, 44·1, centimetres in the first 1, 2, and 3 seconds of its fall. From this we should infer that the acceleration a is 9·8 cm. per second per second. And since $g = 100\,a$, we have $g = 980$ cm. per second per second.

Atwood's machine may be used to verify the various laws we have been describing; for accurate work we need to introduce corrections for the friction and inertia of the pulley.

The pendulum affords us, however, a more exact method of finding g, for it can be shown by means of some simple reasoning, based on the laws of motion, that if T be the time of a complete vibration of a simple pendulum of length l centimetres, then $T = 2\pi\sqrt{\frac{l}{g}}$.

Thus $g = \frac{4\pi^2 l}{T^2}$. Now, the length of a simple pendulum and the time of its swing can be measured very exactly, and from these a value is obtained for g. Thus it is found that the length of a pendulum which makes one complete oscillation in 1 second is about 24·85 cm. Substituting this value above we find the same value for g as previously. By a complete vibration is meant the interval between two consecutive passages *in the same direction* through the lowest point of its swing.

The foregoing results enable us to express in concrete form the force of a dyne, or of a poundal; for since the weight of a body of mass m grammes is mg dynes, we see that the weight of one gramme contains g dynes. Thus, in London the weight of 1 gramme contains approximately 981 dynes. Hence 1 dyne is $\frac{1}{981}$ of the weight of one gramme at London, that is, it is rather more than the weight of 1 milligramme; or, to put it in another way, the weight of 1 milligramme acting on the mass of 1 gramme will produce in 1 second a velocity of rather less than 1 centimetre per second.

And, similarly, if we are working in English units, *i.e.* feet and pounds, we see that since $g = 32{\cdot}2$ feet per

second per second; thus the weight of one pound contains, in London, 32·2 poundals; that is, that the poundal is approximately $\frac{1}{32}$ of the weight of 1 lb., or about the weight of half an ounce. Thus, if the two large masses in Atwood's machine be each $7\frac{3}{4}$ oz. and the rider be $\frac{1}{2}$ oz., then the whole mass moved is 1 lb. while the force acting is the weight of half an ounce, or about 1 poundal. Hence, the velocity generated in 1 second is about 1 foot per second, and the space passed over in the first second about 6 inches.

CHAPTER III

WORK AND ENERGY

WHEN a force moves the point to which it is applied, it is said to do work. The work done is measured by the product of the force, and the distance measured in its own direction through which it moves its point of application. Let us consider shortly these two statements which constitute the definition of work as used in scientific language. Take hold of a weight and lift it vertically upwards, through, say, ten centimetres. You are conscious of a certain amount of muscular effort in overcoming the weight of the body which tended to prevent its being lifted. If you imagine yourself in the same condition as before, the same effort will be required to lift a second equal weight from the floor to the same height, and so on for any number of such weights, while the total effort will clearly be proportional to the total weight lifted, that is, to the total force exerted, since we suppose that just sufficient force to lift the weight, and no more, is used. Now imagine yourself to be again in the same position as at first with reference to the weights, and raise each a second distance

of 10 cm. As much effort is required for this as was previously necessary to raise them the first distance. Thus it requires twice as much to raise them 20 centimetres as was needed for the first rise of 10 cm. If we suppose, in accordance with the ordinary use of language, that in making this effort we are working, the work done, being a measure of this effort, is proportional to the distance the weight is raised. Thus in this instance it accords with our ordinary notions to measure work by the product of the force exerted into the distance through which it is exerted; if the mass is m g. dynes in weight, and it has been raised through h centimetres, the work done is $m\,g\,h$ dyne-centimetre units. It must be noticed that the time taken in performing the work does not come into the measure of work as here defined. Of course we work harder if we lift the weights quickly, but then the total amount of work done in a given time is also greater. If a mass of a kilogramme is lifted a metre, the same amount of work is done, whether it takes a few seconds or many minutes. A man who lifted several kilogrammes a given height rapidly would, no doubt, feel the exertion more than one who performed the same act more slowly; the sensations he experiences depend not only on the total work done, but also on the rate at which it is done. One reason, of course, for this lies in the fact that in rapidly lifting the weight a considerable amount of momentum is given to it in addition to merely raising it, so that probably the work done is actually

greater, but still our sensations alone will not give us a definite measure of work.

We must turn now to the consideration of some of the consequences of that definition. In the first place, it is important to notice that work is not done by a force when its point of application is moved at right angles to its own direction. Thus, suppose we have a body resting on a smooth level surface. The forces acting are the weight of the body and the upward pressure of the surface, and these two are equal and opposite. Suppose, now, the body is moved along the surface; this can be done by the application of the very smallest force, and no appreciable amount of work is required for it. Again, suppose we lift a body from the ground to a height h, but that the second position of the body is not vertically above the first; we may look upon this as lifting the body vertically a height h and then moving it horizontally to its final position. The first operation requires mgh units of work, the second requires no work. Thus the total work done is mgh units. Again, suppose that a force of F dynes is acting on a body at A in a direction A B (fig. 1), and that the body is moved from A to C. Draw C B at right angles to A B; then we consider the motion from A to C as made up of a displacement from A to B, in which the force does work, given by F × A B, together with a displacement from B to C, in which F does no work. Thus the work

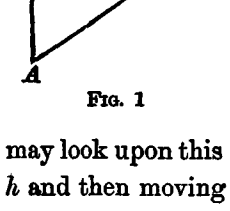

FIG. 1

done by the force F in the given displacement is F . A B. The distance A B is the displacement of the point of application of the force 'measured in the direction of the force.' Of course the displacement A C would not take place under the action of the force F alone. Some other force must have been applied to the body to produce it, and F . A B only measures the work done by the force F.

Again, if a force moves the body to which it is applied in its own direction, it is said to do work on the body, while, when a body is moved by some external agent in a direction opposite to that of the force, it is said that work is done against the force; thus, if I apply to a body a vertical force slightly greater than the weight of the body, and so lift it, the force thus applied does work on the body against gravity, while if a body is allowed to fall vertically under the action of its weight or gravity, then gravity does work on the body. The work done in all cases is measured in terms of some unit of work depending on the units of force and space. And a unit of work is done when unit force moves its point of application through unit distance. The C.G.S. unit of work is called an 'erg'; it is the work done by a dyne in moving its point of application 1 centimetre. Thus, if a mass m grammes be raised a height of h centimetres, since the weight of m grammes is mg dynes, an amount of work equal to mgh ergs has been done on the body against gravity. The corresponding English unit of work is the foot-poundal, which is the work done by a poundal in moving its point of

application 1 foot. Thus, since the weight of m pounds is mg poundals; if a mass of m pounds be raised a height of h feet the work required to do this is mgh foot-poundals. It must, of course, be remembered that in this last expression g is measured in feet per second per second, and has, therefore, approximately the value 32·2, while in the expression in ergs g is measured in centimetres per second per second, and has the value 981. Thus, to raise a kilogramme to the height of 1 metre requires $1000 \times 981 \times 100$ ergs of work, while to raise a mass of one *stone* or fourteen pounds through a yard requires $14 \times 32{\cdot}2 \times 3$ foot-poundals.

A third unit of work in common use is the foot-pound. This is the work done in raising 1 lb. a distance of 1 foot against gravity. Since the weight of 1 lb. varies from point to point on the earth, the foot-pound has slightly different values in different places. Since the weight of 1 lb. contains g poundals, a foot-pound contains g foot-poundals. Hence the work done in lifting a mass of m lbs. through h feet is mgh foot-poundals, or mh foot-pounds. In measuring work in foot-pounds we must remember that we are adopting the weight of 1 lb. as the unit of force, and be consistent throughout; *e.g.* we cannot straightway without further change apply the equation $F = ma$, for this implies that the unit of force produces unit acceleration in unit of mass; and if the mass of 1 lb. be our unit of mass, and the weight of 1 lb. our unit of force, this condition is not satisfied.

The rate at which an agent can work, *i.e.* the amount of work it can do in the unit of time, usually one second, is called the 'power' of the agent. Thus, 'power' is the rate at which work is done, and a machine possesses a unit of power when it can do a unit of work in 1 second. Thus, on the C.G.S. system a machine which can do 1 erg per second has unit power. A very usual unit of power is the horse-power; this is the rate of work of an agent doing 550 foot-pounds of work per 1 second.

One difficulty which hinders the general adoption of the centimetre-gramme-second system of units is the fact that they are all very small. A dyne is an extremely small force; if we wish to express in dynes any ordinary force it will probably require an extremely large number, possibly many millions, and this is inconvenient. We can, of course, measure in kilodynes, or thousands of dynes, in megadynes, or millions of dynes, and so on. Special names have, however, been given to certain multiples of the units of work and power, and these are now generally adopted.

Thus, a 'joule' is ten million C.G.S. units of work, while a 'watt' is the power of 1 joule per second.

Hence 1 joule $= 10^7$ ergs.
1 watt $= 10^7$ ergs per second.

These, then, are the practical units of work and power on the C.G.S. system. If we remember that the weight of a gramme is rather less than 1,000 dynes, a

joule will be rather more than the work done in raising a kilogramme through 10 centimetres. The work done in raising 1 kilogramme 10 centimetres is really 9·8 million ergs, while a joule is 10 million ergs.

We can show easily from the above that a joule is rather less than $\frac{3}{4}$ (more exactly ·737) of a foot-pound, and a watt rather less than $\frac{3}{4}$ foot-pounds per second. Thus a horse-power is 746 watts.

When a force does work on a body or system of bodies, it is said to increase the energy of the system, and the increase of energy is measured by the work done by the force. Or again, conversely, when a body, or system of bodies, is in a condition to do work, it possesses energy, and the energy possessed is measured by the work which can be done; the energy of a system, then, is its capacity for doing work, and is measured either by the work which it can do, or, supposing for the present that there are no forces of the nature of friction to be considered, by the work which must be done on it to bring it from a condition in which it has no energy to its given state.

Thus, consider a mass which has been raised a height h from the ground against gravity. To do this has required an expenditure of mgh units of work; thus the energy of the weight has been increased by mgh units —ergs, or foot-poundals, according to our fundamental units of mass and space. We have here supposed that the weight has been raised so slowly that it arrives at the top without appreciable momentum. Suppose, now,

that the mass is connected by a fine string passing over a frictionless pulley with another slightly lighter mass on the ground. If we let it go it will fall, and in falling will draw up the other mass, thus doing work and losing energy, which will be gained by the other mass. When at a height h the first mass, in consequence of its position with reference to the earth, possesses energy; this energy, depending as it does on the configuration or relative position of the bodies of the system, is called potential energy. But a body may possess energy in other ways than the above. A cannon-ball in motion has a large store of energy, and this depends, not on its position, but on its mass and velocity. Energy of this kind, depending on the motion of the body which possesses it, is called kinetic; it is measured by the work which must be done on the body to bring it to a state in which it has no kinetic energy; or, conversely, to bring it from a condition of no kinetic energy to the given state.

Now, suppose the mass of the body is m grammes and its velocity v centimetres per second; suppose also this velocity could be produced in it by a force of F dynes acting for t seconds, and, further, that the acceleration produced by this force is a centimetres per second per second, while the distance the body moves in t seconds is s centimetres.

The momentum of the body is mv, and the force required to give it this momentum in t seconds is $\frac{mv}{t}$

dynes. Again, since v is the velocity at the end of t seconds and the velocity has increased uniformly from rest, the distance traversed is $\frac{1}{2} v t$ centimetres. The work done is measured by the product of the force and the distance through which it acts; thus, in this case the work done is $\frac{m v}{t} \times \frac{1}{2} v t$, and this is $\frac{1}{2} m v^2$ ergs.

But this work measures the kinetic energy of the body.

Thus the kinetic energy is $\frac{1}{2} m v^2$ ergs, the mass being in grammes and the velocity in centimetres per second. If the mass be measured in pounds and the velocity in feet per second, the energy is, of course, $\frac{1}{2} m v^2$ foot-poundals, and, since a foot-pound contains g foot-poundals, the kinetic energy in foot-pounds is

$$\frac{1}{2} \frac{m}{g} v^2.$$

The above result can be exhibited in symbols thus:

We have $\quad F = m a, \; s = \frac{1}{2} \frac{v^2}{a}$

$\therefore$ Kinetic energy $= F s = m a \times \frac{1}{2} \frac{v^2}{a} = \frac{1}{2} m v^2$.

Thus, a bullet whose mass is one third of an oz., and which is moving with a velocity of 500 feet per second, will have $\frac{1}{2} \times \frac{1}{3 \times 16} \times (500)^2$, or about 2,604 foot-poundals of energy. This is approximately equal to

$\frac{2,604}{32}$ foot-pounds, so that the energy of this bullet would suffice, if it could be properly applied, to raise a pound weight about 81 feet, or 1 cwt. through about 9 inches.

Thus up to the present we have recognised two forms of energy—potential and kinetic. Let us consider the transformations which may occur from the one form to the other in some simple cases. Take first that of a mass m, which has been raised to a height h and then been allowed to fall freely. Just at starting its energy is all potential, and is measured by $m g h$; its velocity being zero, it has no kinetic energy. Let v be the velocity with which it reaches the ground; at that instant its energy is all kinetic, and is equal to $\frac{1}{2} m v^2$. Its potential energy is now zero, for that depends on its height above the ground. Its energy has been transformed from the potential to the kinetic form. Again, since the velocity v is generated by falling a height h we can show that $v^2 = 2 g h$. Thus $\frac{1}{2} m v^2 = m g h$, or the kinetic energy at the bottom is equal in amount to the potential at the top; there is neither loss nor gain of energy in the transformation. It may be surmised from the above that, since the energy is the same at the top and bottom, it has remained unchanged in amount during the fall. And this on investigation proves to be true; for consider the body after it has fallen part way, it has lost a certain quantity of potential energy,

viz. that due to the distance it has fallen, but at the same time it has gained an exactly equal amount of kinetic, so that the total energy is the same. For suppose its velocity is v after falling a distance z, then the loss of potential energy is $m g z$, while the gain of kinetic is $\frac{1}{2} m v^2$; and since $v^2 = 2 g z$, these quantities are equal, or the total energy is unchanged. We may put the same result in the following manner: after falling a distance z, its height above the ground is $(h - z)$; thus its potential energy is $m g (h - z)$, while its kinetic energy is $\frac{1}{2} m v^2$. Thus its total energy at this point of its fall is $m g (h - z) + \frac{1}{2} m v^2$, but $\frac{1}{2} m v^2 = m g z$. Thus the energy at any point of its fall $= m g (h - z) + m g z = m g h = \frac{1}{2} m v^2$. Thus in this fall there is neither loss nor gain of energy, only transformation from potential to kinetic. This constitutes our first example of the great principle of the conservation of energy, one of the guiding laws of Physical Science. A mass of 1 lb. raised to the height of 10 feet has in consequence 10 foot-pounds of energy; if allowed to fall it will continue until the moment it reaches the ground to have 10 foot-pounds. On striking the ground it appears to lose both its potential and kinetic energy. In reality, another transformation takes place here, and this we shall soon consider.

If instead of dealing with a weight falling freely we consider a pendulum, we have here repeated transformations of energy, from potential only when at the highest point of the swing, to kinetic when at the lowest, and

then back to potential as the bob rises on the other side, the whole energy remaining constant except for the frictional effect of the air resistance and at the points of support.

Potential energy in the cases we have been dealing with is due to the position of the mass m relatively to the earth, and to the force of attraction between it and the earth, but this, of course, is not the only source of potential energy; let us consider some others. Energy is required to drive a clock or watch, to give the necessary momentum to the wheels and hands, and to overcome the friction at the pivots. This energy comes from the work which is expended in winding up the watch. The free, unstrained position of the mainspring is its uncoiled state. Work is required to wind it up, and the work done is stored as potential energy in the spring when wound. The mechanism of the watch converts this into the kinetic energy of the hands and wheels, and the watch stops when the whole store of potential energy is thus converted. Or again, a bent bow is a source of potential energy. Work is done in bending the bow, and remains as potential energy when the bow is bent; when the string is released this potential energy becomes transformed mainly into the kinetic energy of the arrow.

Again, many chemical combinations are stores of potential energy, like the coiled mainspring, or the bent bow. Such, for example, is gunpowder; to apply a light is like loosing the bowstring, and an explosion is the

result; the potential energy of the combination thus released is transformed in the barrel of the gun to the kinetic energy of the bullet. In all these instances we have illustrations of an important natural law. The systems all tend to place themselves in conditions in which their potential energy is as small as possible; the suspended weight falls to the ground; the coiled watch-spring uncoils and drives the watch; the bent bow straightens and propels the arrow; the burnt gases which result from the explosion of gunpowder have much less energy than the powder; and in these various cases the universal tendency of potential energy to become transformed to the kinetic form, and of the various systems to settle down to the condition of a minimum of potential energy, is made use of by man for his own purposes.

Let us now look a little more closely into the case of the bullet. The powder, as it explodes, exerts force on the bullet, propelling it along the barrel. Let us suppose that the force exerted at any moment is F units. Probably F varies from instant to instant, as the explosion continues. We will, for simplicity, take an average value for it, and see what conclusions we can draw from it. Let the length of the gun barrel be s centimetres, and suppose it takes the bullet t seconds to traverse it. Let m be the mass of the bullet, and v its velocity at the mouth of the barrel.

Then the work done by the force shows itself in the kinetic energy of the bullet. The force F has acted

through the distance s and has, therefore, done $\mathrm{F}\,s$ units of work, the kinetic energy is $\frac{1}{2}mv^2$, and hence we have

$$\mathrm{F}\,s = \tfrac{1}{2}mv^2.$$

Again, the force F has generated momentum mv in time t, and therefore we have

$$\mathrm{F}\,t = mv.$$

If we know the mass of the bullet, its velocity, and the length of the barrel, we can find F and t. Thus, taking the data of the example on p. 32, we have

$$m = \tfrac{1}{8} \text{ oz.} = \tfrac{1}{128} \text{ lb.}; \quad v = 500 \text{ feet per sec.}$$

Hence, supposing s, the length of the barrel, to be 3 feet, we have $3\,\mathrm{F} = 2{,}604$ foot-poundals.

Thus $\mathrm{F} = 868$ poundals.

Also, $t = \dfrac{mv}{\mathrm{F}} = \dfrac{500}{48 \times 868} = \cdot 012$ seconds nearly.

Again, while the powder has exerted this force on the bullet, the bullet has exerted an equal and opposite force on the powder, and through it on the gun, causing it to recoil, and the momentum of recoil thus generated in the gun is exactly equal to that of the bullet. The mass of the gun being greater than that of the bullet, the velocity of the gun will be proportionately less than that of the bullet.

There is, however, an important difference between the slow motion of a heavy body like the gun, and the rapid motion of the bullet, even when the momenta of the two are the same. Let us consider the force which is required to bring either body to rest in 1 second.

It is the same for the two, and is measured by the momentum. Now, a body which is initially moving with velocity v, and which is brought to rest in 1 second by a constant force, will in that 1 second move a distance equal to $\frac{1}{2}v$; suppose the mass of the gun to be 10 lbs., if it were hung freely it would recoil back with a velocity of $\frac{500}{480}$, or rather more than 1 foot per second. Suppose it is brought to rest in 1 second by the application of the proper force, it will in that time move rather more than 6 inches; since the bullet has the same momentum as the gun, the same force will bring it to rest in 1 second, but in that time it will have moved 250 feet. Thus the penetrating power of the bullet, as measured by the distance it can go before being stopped, is much greater than that of the gun. The energy which propels the bullet is derived from the potential energy of the powder, and it might be shown experimentally, though the experiments would be difficult, that the potential energy of the powder was reproduced in the energy of the bullet and gun, and in one or two other forms which we shall go on to consider.

Now let us return to the falling weight. We have seen that just as it reaches the ground its kinetic energy is equal to the potential energy with which it started; but what has happened when it has struck the ground and remains at rest? It has apparently lost all its energy, it is not in motion, so its kinetic energy is zero; it is on the ground, so its potential energy is zero; still, certain changes have occurred. If examined with sufficiently

delicate instruments, it will be found that both the weight and the ground are made perceptibly warmer by the blow. The fall has produced sound and possibly also light; for all these effects energy is required, and the rise of temperature, the intensity of the sound or of the light flash, can be expressed in terms of energy or work. When all these and possibly other effects which may have been produced are so expressed, it is found that the sum total of the energy is still the same as before; the energy is still conserved, though changed in form. Of course the above is merely a brief statement, and will require careful consideration in detail; let us take the case of heat first, and consider the evidence for the statement that heat and energy are mutually interchangeable. This rests on the facts that there are many ways in which heat can be produced from mechanical work, and that in all these ways the heat produced bears a fixed ratio to the work used for its production. Taking as the unit of heat the heat required to raise 1 lb. of water 1° Centigrade, Dr. Joule showed that at Manchester about 1390 foot-pounds of work are required to produce 1 unit of heat.

If we take as the unit of heat the heat required to raise 1 gramme of water 1° Centigrade, then the number of units of work required is about 41·6 million ergs, and this we know is 4·16 joules. That heat can be produced from work is shown in a variety of ways; the boy who rubs a button on his coat-sleeve and burns his neighbour's hand with it has produced heat from work.

In all cases of friction work is done and energy changes its form, showing itself as heat. Perhaps the earliest direct experiments on the subject were those of Count Rumford at the end of the last century. The results were communicated to the Royal Society of London in 1798 in a paper called 'An Enquiry concerning the Source of the Heat when it is excited by Friction.' He noticed the immense amount of heat produced by boring a brass cannon, and set himself to measure it by immersing the cannon and borer in water; he found thus that he was able to boil the water, the mass of which was 18·77 lbs., in two hours and a half. Making allowance for the heat used in raising the temperature of the cannon, borer, and other parts of his apparatus, he found that enough heat had been produced to raise the temperature of 26·58 lbs. of water from its freezing point to its boiling point. According to the theory of the nature of heat held at the date of this experiment, the heat produced was due to the disintegration of a part of the brass by the borer. It was supposed that heat was an imponderable fluid called caloric, and that a body became heated by acquiring more of this caloric; while, further, it was maintained that the finely-divided brass could hold less caloric than the same mass of brass in the continuous state; the surplus caloric was thus given to the water and raised its temperature. Rumford pointed out the difficulty of contending this, showing that 'the source of heat generated by friction in these experiments appeared inexhaustible,' concluding, there-

fore, that heat cannot be a material substance, and that thus it appeared 'to him to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated by these experiments except it be motion.' Instead of motion we should now say energy. But Rumford's experiments were not conclusive. He did not show that, as regards its thermal properties, his powdered brass was in the same state as the solid brass; this might have been done by submitting equal quantities of the two to the action of nitric acid, when he would have found that the final results were the same, and that in attaining to the final condition equal quantities of heat were produced. It was still open, however, to the adherents of the caloric theory to maintain their view.

Another passage in Rumford's paper may be quoted:

'One horse would have been equal to the work performed, though two were actually employed. Heat may thus be produced merely by the strength of a horse. . . . But no circumstances could be imagined in which this method of producing heat would be advantageous, for more heat might be obtained by using the fodder necessary for the support of the horse as fuel.'

Rumford's experiments were followed by those of Sir Humphry Davy, who melted wax by rubbing together two pieces of metal surrounded by ice in the exhausted receiver of an air-pump. He showed thus that the heat did not come from the ice, for instead of growing colder as it should have done had it given up

some of its caloric to the metal and wax, it was, if anything, slightly melted, and had, therefore, received heat; but he did not draw the correct inference from this result, viz., that heat is not a material but one form of the energy originally stored in the clock-work which moved his piece of metal.

The earliest of the long series of experiments by which Joule finally settled the question as to the identity of heat and energy, was published to the world in a paper, which he communicated in 1843 to the British Association meeting at Cork. The work was continued during the succeeding years, and in 1849 a definite result was communicated to the Royal Society in the following terms:

1. That the quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of energy [1] expended.

2. That the quantity of heat capable of increasing the temperature of 1 lb. of water (weighed in vacuo and taken at between 55° and 60°) by 1° Fahr., requires for its evolution the expenditure of an amount of mechanical energy represented by the fall of 772 lbs. through the space of 1 foot.

Since 1° Fahr. is equal to $\frac{5}{9}$ of 1° Cent., it will require $\frac{5}{9} \times 772$, or nearly 1390, foot-lbs. of work to raise 1 lb. of water 1° Cent.

[1] In his paper Joule uses the word 'force,' not energy, but he has previously defined force as meaning what we now call energy. I have therefore, in the above statement, replaced force by energy.

This most important result of Joule's, perhaps the greatest discovery in Physical Science since the time of Newton, has been fully confirmed by other experimenters using various methods. Joule himself, in a paper read before the Royal Society in 1878, gives as his final result at sea-level at Greenwich referred to water weighed in vacuo between 60° and 61° Fahr. the number 772·55. This becomes in terms of the Centigrade scale 1390·6, the water being at a temperature of 15°·8 C. Other experiments have shown that if the water were at 0° C., it would require rather more heat, in the ratio of 1·00089 to 1, to raise its temperature the same amount, and this gives as the value 773·24 or 1391·8. To reduce to ergs per gramme degree Centigrade we have to multiply by 981·17, for this is the value of g, and by 30·48, since there is this number of centimetres in a foot. The result is $4{\cdot}1624 \times 10^7$ ergs. Thus finally we may say that 4·1624 joules raise 1 gramme of water at 0° C. in temperature 1° C. This quantity, the number of units of work required to produce 1 unit of heat, is called Joule's equivalent, or the mechanical equivalent of heat, and is generally denoted by J.

This result is less by one part in 550 than that found by Prof. Rowland in 1879 at Baltimore, when corrections are introduced for the difference in the thermometers.

Joule's best results were obtained from experiments on the stirring of water. A known mass of water is

contained in a vertical cylinder of metal. Within this a shaft carrying paddles revolves. Fixed vanes are

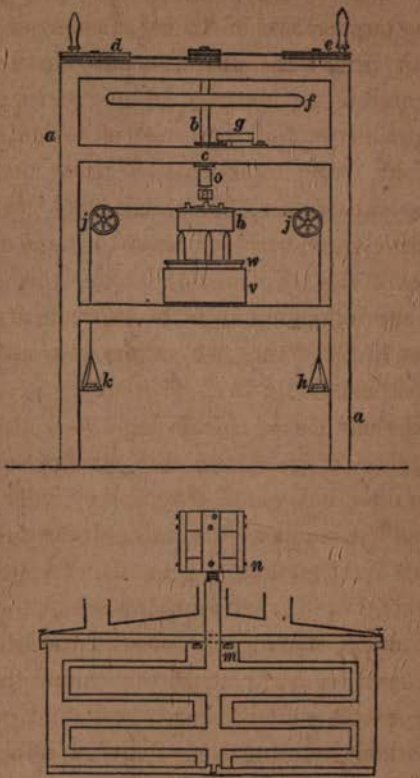

FIG. 2

secured to the sides of the cylinder, and the moving paddles rotate between these. When the shaft is turned, the temperature of the water is raised by the

friction between the paddles and the water, and the rise of temperature is measured by very delicate thermometers. Allowances are made for the heat necessary to raise the temperature of the calorimeter, and for that radiated out from the apparatus, and hence the total heat generated is calculated. In one series of Joule's earlier experiments, the total rise of temperature was 0°·5632 Fahr. The total mass of water used, allowing for the calorimeter, &c., was 13·924 lbs., so that the heat developed was sufficient to raise a mass of 13·924 × ·5632 lbs. of water 1° Fahr.; this comes to 7·842 lbs. approximately. In the experiments of 1878 the rise was 5°·22 Fahr., the corrected mass of water being almost exactly 12 lbs.

As to the method of calculating the work done; in the experiments of 1849, two thin strings were wound in opposite directions round a wooden cylinder attached to the axis of the paddles; the ends of these strings were carried over light pulleys (*j j*, fig. 2), and attached to equal weights, *k k*. On releasing the axle *b*, the weights slowly descended, turning the paddles and heating the water. When they had reached the ground the strings were, by a simple contrivance, *d e*, rewound on to the cylinder without rotating the paddles, and the experiment was repeated 20 times. The force producing this effect, making corrections for the friction at the pulleys and other small loss, was equal to the weight of 57·62 lbs., and the total fall, allowing for the velocity with which the weights reached the ground, was

105 feet, giving 6050 foot-pounds as the work done. To this about 17 foot-pounds had to be added for work developed by the elasticity of the string which carried the weights; thus the total energy used was 6067 foot-pounds. Hence the energy required to raise 1 lb. of water 1° Fahr. is $\frac{6067}{7 \cdot 842}$ foot-pounds.

Thus the result of this experiment is 773·6 foot-pounds.

This is only one, however, of the long series of experiments made in 1849.

In 1878 the method of finding the work absorbed was somewhat different; the cylinder was itself free to rotate, and if not restrained would turn in consequence of the friction between itself and the water; the work done on the water in preventing this rotation was determined, and from it the result already given was deduced. A somewhat similar method of obtaining the work was used by Prof. Rowland.

These experiments, then, triumphantly establish the fact that heat is a form of energy, and that a definite amount of energy, if used in producing heat, produces a definite amount of heat. This constitutes a law known as the First Law of Thermodynamics, the name given to the mechanical science of heat.

We now see what has become of the energy apparently lost in some of the cases which we have already discussed. The weight that has fallen to the ground has lost its potential energy and its kinetic

energy of visible motion; nearly the whole has been transformed into heat, and the temperatures of the weight and the ground on which it fell have been correspondingly raised. A pendulum set swaying gradually comes to rest because of the friction at the support and the resistance of the air. The energy of the pendulum is transformed in part into heat at the support, in part it is absorbed in setting in motion the air surrounding it, and this part in the end becomes gradually transformed into heat. In neither case is energy really lost; its form is changed, but its amount remains the same.

CHAPTER IV

FORMS OF ENERGY

BUT energy is capable of taking other forms besides those just mentioned. Let us just consider very briefly some of these, and first, perhaps, should come the energy of chemical separation.

All material substances are made up of certain chemical units called elements, which combine in certain definite proportions to form these substances. Now, for reasons which are quite unknown to us, a given element will combine more readily with some of the elements than it will with others; this attraction between the various elements is a source of energy. Let us return for a moment to Newton's law of attraction between particles of matter, and suppose we consider a large space throughout which an immense number of particles of fine dust are distributed; each particle pulls and is pulled by all the others, and the mutual attractions of these particles constitute a store of potential energy. The resultant effect of these numerous forces is that the particles are pulled closer together and condensed: they thus lose some of their

potential energy, gaining an equal amount of kinetic, but any given particle we may be watching would soon be seen to strike one or other of its neighbours; in consequence of this impact, except under special conditions which are not at all likely to hold, the kinetic energy of visible motion is reduced; some of it is transformed to heat, and in the end the mass becomes a solid nucleus with its potential energy of position much reduced, but its temperature greatly raised. The chemical affinities of the elements seem to act in a somewhat similar way to this, only the forces between the various atoms or molecules of a substance are extremely small except where those molecules are very close together.

Thus, common soda is a compound of the three elements, sodium, carbon, and oxygen, while the substance called muriatic acid contains hydrogen, chlorine, and water, the water being made up of oxygen and hydrogen. In the first the sodium, carbon, and oxygen are held together by the chemical affinities between them; in the second the hydrogen and chlorine stick together and are mixed up with the water. Now, the bonds which connect the sodium with the carbon and oxygen are not very strong, neither are those which bind together the hydrogen and chlorine; on the other hand, the affinity between sodium and chlorine is a very powerful one. Suppose, then, we mix these two substances together; we find a great change, violent effervescence takes place, and a gaseous substance is given off from the liquid. This gaseous substance is found to contain

carbon and oxygen, though not in the same proportion as they were in the soda, while in the liquid left behind we find water and common salt. Now, common salt is a compound of chlorine and sodium, the sodium has been torn away from the carbon and oxygen and the chlorine from the hydrogen to form it. Besides this, the hydrogen has taken some of the oxygen from the carbon to form water. The substances have thus lost some of the potential energy of chemical separation which they possessed before being mixed; this has been transformed into heat, and the potential energy has been reduced; the resulting compound is more stable than the elements of the mixture before the combination took place, and, in accordance with our general theorem, the final state is one in which the potential energy is, under the existing conditions, a minimum.

As an illustration of this process, let us suppose we had two walls each held together by very weak mortar, the one of blue and red bricks, the other of black and white, say, arranged in alternate rows. These may represent two chemical combinations. Let there be also a very strong attraction between the blue bricks and the white when they are close enough together. Suppose, further, that it is possible to bring the two walls so close together that this attraction can act; the result may clearly be that all the blue and white bricks may be pulled out of place and stick firmly together, while the red and black are thrown down in confusion.

A new substance made of the combination of the blue and white bricks is formed, and this of course is mixed up with the red and black bricks; potential energy is lost and heat produced.

In the phenomena of combustion we see another case in which the potential energy of chemical separation becomes transformed into heat. The bond that links together carbon and oxygen when brought into close proximity is a very strong one, but somehow, at ordinary temperature, the molecules of carbon and oxygen cannot, as it were, get close enough together for this affinity to come into play. By raising the temperature we enable the action to take place. Now, coal or charcoal consists largely of carbon, and there is plenty of oxygen in the air. When we apply a light to a piece of coal, we raise its temperature, and that of the air, so that a small portion of the carbon can combine with the oxygen about it. A considerable amount of potential energy is transformed into heat, and the temperature of the rest of the coal is raised so that the combustion can proceed. The coal is burnt. The amount of heat produced by the combination of various substances with oxygen has been carefully determined, and it has been found that enough heat is produced by the burning of 1 gramme of hydrogen to raise about 34,000 grammes of water 1 degree, while the combustion of a gramme of wood charcoal will raise about 8,000 grammes of water 1 degree.

In this case we obtained the oxygen from the air.

Now, nitre is a substance which is composed of nitrogen, potassium, and oxygen held very loosely together, and it contains a large store of oxygen. If we bring it into close contact with charcoal and then raise the temperature to start the combustion, the combination takes place, and the same gas, called carbon dioxide, is formed. This occurs when gunpowder is fired; for gunpowder consists of a mixture of nitre and powdered charcoal with a little sulphur, which helps the process of burning. If the combustion takes place in a small space, like the barrel of a gun, the heat produced by the combination raises the temperature of the gas and so causes a great pressure. This is relieved by the bullet giving way and being expelled violently from the gun. As the gas expands, its temperature, as we shall see shortly, falls, the heat energy being transformed into the kinetic energy of the bullet. Thus the potential energy of the chemical combination of the carbon and oxygen is changed into the kinetic energy of the bullet. Suppose that the bullet strikes an iron target; it is flattened out and heated, we hear the thud on the target, maybe we see a flash of light. Some of the energy of the bullet is used in producing this sound and light, for sound is due to the small vibratory motion of the molecules of air, and energy is transmitted by these moving particles to our ears, where, acting through the tympanum or drum of the ear on the nerves of our auditory apparatus, it gives us the sensation of hearing; while light also travels in waves through a medium called the

luminiferous ether which pervades all space and affects the nerves of our eyes.

To illustrate the process by which sound is conveyed, consider a number of balls hung in a row by strings of equal length, and suppose that when at rest they do not quite touch one another; take the end ball and raise it a little, keeping the string stretched—this gives it potential energy—then let it go. It falls like the bob of a pendulum, its energy becoming kinetic, and after striking the second ball rebounds, the second ball moves on, strikes the third and rebounds, and so on. The balls continue to oscillate about the positions they initially occupied, but part of the energy originally conferred on the first ball is conveyed along the row. Now imagine this process continued, the first ball being caught as it swings back from its first impact, raised to the same height as previously and again let go; more energy will travel along the row; the energy supplied to the first ball is conveyed to the far end of the row in the form of a wave. These balls represent in a very rough, imperfect manner the particles of air which convey a wave of sound. The sounding body is in a state of vibration and corresponds to the hand which keeps the first ball in motion, the air particles near it are thereby forced to vibrate and act upon those in contact with them, and so the energy is transmitted onwards.

But energy takes other forms than these. Scatter some iron filings on the table and bring a magnet down towards them from above; the filings are attracted and

rise up to the magnet; work is done on them against gravity, and to do this work requires an expenditure of energy from somewhere. The magnet is pulled down by the filings. If we suppose it suspended from one pan of a balance more weights will be needed to preserve equilibrium when the filings are below than are needed when the filings are removed. The magnet and the filings are a system possessing potential energy due to magnetisation. Or again, rub a piece of ebonite or sealing wax with fur and then hold it over some small bits of thin paper or near a light pith ball suspended by a thin thread. The paper and the pith ball are at first attracted to the ebonite and fly up to it; they are then repelled away from it. The ebonite has been electrified by friction and has in consequence acquired energy. We could show that while the ebonite attracts the fur with which it has been rubbed it repels a second piece of ebonite which has been treated in a similar manner. Owing to this attraction work has to be done to separate the ebonite and the fur after friction, and this energy, together with some of that expended in the rubbing, is transformed into the energy of electrical separation.

Once more, place a piece of pure zinc and a piece of copper in water in which there is a little sulphuric acid or oil of vitrol, and keep the plates apart. If the zinc be pure nothing in particular happens; now connect the plates with a wire outside the liquid. It will be observed that the zinc is slowly dissolved away, and that

bubbles of gas, which can be shown to be hydrogen, collect on the copper. If the zinc be impure, bubbles will be formed on it before the connection with the copper is made and will continue to form afterwards. Bring the wire near a compass needle. In general the needle will be deflected; there is a current of electricity flowing in the wire and it has become in consequence a source of energy; if the current is sufficiently strong it will be found that the wire gets hot as the current passes, it may become red or white hot, as in the incandescent electric lamps; part of the energy supplied then takes the form of light. The simple battery just described will not supply sufficient energy for this. If, again, the current be passed through a coil of wire which is hung up by a thread so that it can take any position readily, it will be found that the coil sets in a definite manner; another coil in which a current is passing or a magnet will cause it to move, it possesses a definite amount of energy so long as the current is continued; this energy is derived from the chemical changes which go on in the battery cell and can be measured in terms of these changes.

It can be shown that in none of these cases is there any change in the total amount of energy in the system; the changes are merely in the form assumed by the energy. Hence we assert that if we have any system of material bodies, and if no energy is supplied to that system from without and no energy is withdrawn from the system, then the total amount of energy in

the system is constant though the forms assumed by the energy change. This is the principle of the conservation of energy in its general form.

The investigation of the changes which take place in the forms of the energy of a material system is the subject matter of Physics. To treat all the branches of physics in detail would require a book for each. We are at present concerned with some general laws and properties common to matter in its several forms.

CHAPTER V

FORMS OF MATTER—SOLIDS AND FLUIDS

In our opening chapter we have spoken of matter as that which occupies space; let us now consider a little more closely the forms which can be assumed by portions of matter and see how we can classify them. Such a consideration will lead us to discuss some terms in ordinary use to describe states or properties of matter. Consider a lump of iron or steel; if force is applied to it, it yields very little indeed. If we attempt to compress it into a smaller volume, we find it offers great resistance. A very great force can change the volume by only a very small amount. If we attempt to change its shape, the result is much the same. A large force produces a very small effect. We may take a ball of iron and squeeze it in a vice; it will yield a little, and, of course, when squeezed it will be flattened out of the spherical shape, but the change will not be great, the iron or steel is only very slightly compressible, and it is also very rigid. Rigidity is the term used to denote the resistance which a body offers to forces tending to change its shape. A perfectly rigid

body would be one which would never change its shape, however great might be the forces acting on it. No body is perfectly rigid; but iron, steel, glass, and many other substances have the property in a marked degree. If the forces which we suppose are acting on the body be not too great, the body will recover its original size and shape when these forces are removed. Elasticity is a somewhat vague term used to denote this property of a body.

An elastic solid is one which has a perfectly definite form and volume when free from the action of external forces. Perfectly definite changes in form and volume, strains as they are called, are produced by the action of given forces, and, further, provided these forces lie within certain limits, when they are removed the body recovers its original state. In bodies like iron, steel, glass, &c., which are without crystalline structure, the elastic properties are defined by (1), the resistance the body offers to compression, and (2), the resistance it offers to change of shape. In common parlance, however, a body which readily recovers its size and shape, after considerable deformation, is said to be highly elastic; it would be more accurate to say of such a body that the limits are considerable within which the strains may lie without impairing the elastic properties of the body, or preventing it from recovering its form and volume when the stresses are removed.

Let us now consider a substance like jelly or india-rubber. We can easily squeeze it out of shape; an

india-rubber ball may be flattened considerably with very moderate pressure. When this is done, however, it will be found that the volume is not much altered. Suppose we take such a ball, and imagine equal forces applied to each point of its surface, all pressing inwards and tending to reduce its volume, though not affecting its spherical shape; it will be found that a very considerable force will not make the ball much smaller, or, if we put it in a vice and squeeze it, we shall find that the ball will extend so much in other directions that its volume is very little changed. India-rubber opposes great resistance to change of volume, but very little resistance to change of form; it has very little rigidity, but at the same time it is comparatively incompressible. Or again, take a substance like beeswax or putty; these have some rigidity, they retain their form, and it requires a certain, though very small amount, of force to make them change it. They are, however, very easily either compressed, or altered in shape, and that permanently; their rigidity, and the resistance they can offer to compression are both extremely small; a very trifling force is sufficient to strain them beyond their elastic limits.

All these substances which possess rigidity are classed together as SOLIDS.

Bodies which at all points of their substance have the same properties exactly are said to be homogeneous; such, for example, are glass, brass, or iron, water, or other fluids, and many other substances. If we cut two small equal cubes from different parts of such a substance,

taking care that the corresponding edges of the cubes are parallel, they will resemble each other in every respect, and could be interchanged without altering the properties of the substance in any way. They will be equal in mass, and all their elastic and other properties will be alike. If this is not the case, the substance from which the cubes are cut is heterogeneous, that is, composed of different materials in different parts. A piece of conglomerate rock is an example of a heterogeneous substance. It is probable that if the cubes cut out even from a homogeneous body as described above, were sufficiently small they would differ somewhat from each other. This might easily be the case if they were comparable in size with the molecules of the body. One cube might conceivably be taken so as to include only one molecule, while the other included parts of two or more. If we could render the molecular structure visible, in many cases a want of regularity would appear. In describing the various properties of bodies, we shall, as a rule, suppose them to be homogeneous.

Homogeneous bodies, again, may be subdivided into two classes. In the first of these the properties of the body are the same in all directions about any point. Bodies of this class are called isotropic. In such bodies it is not necessary that the cubes cut as above should have their edges parallel; any two cubes of equal size, cut anyhow from the body, have identical properties; or, to put the same fact rather differently, one small cube can be conceived of as being cut out of the body, and

replaced, but with its edges turned into different positions from those which they previously occupied, without altering the nature of the substance at all. The molecules of such a body are arranged entirely at random, but in such a manner that if we start from any point and move a given distance, we shall pass exactly the same number of molecules in whichever direction we go. The second class of homogeneous bodies is said to be æolotropic. If we take two parallel directions in such a body, the properties of the body in these two directions are the same; but if the directions be not parallel, then the properties are different. Two equal cubes cut out from the body will exactly resemble each other, if the edges be parallel, and one cube can be replaced by the other, provided this parallelism be retained. We may conceive of the body as having its molecules more closely packed in some directions than in others; or again, if we prefer it, that the molecules are not spherical in shape. Crystals of quartz, Iceland spar, or arragonite, and most other crystalline substances, are specimens of æolotropic bodies. The elastic properties of such bodies are different in different directions, so are their electric properties, while light travels through them with a velocity which depends on the direction.

If a substance has a shape of its own which it can permanently retain without being in any way constrained to do so, and which cannot be altered except by the action of an appreciable force, then it is a solid.

Substances of which this assertion cannot be made,

which have no definite shape, but accommodate themselves to that of the vessel in which they are placed, and which oppose no appreciable resistance to the action of forces tending merely to change their shape, are called fluids.

A FLUID is a substance which has no rigidity. Thus, water is a fluid; if you pour it into a vessel it takes the shape of the vessel; if it is poured on to a flat surface it spreads out in a thin film over the surface. In some cases small quantities of the water will collect and stand in drops on the surface; this is due to capillarity, which will be considered in detail later. In the form of ice, water substance is a solid; it has some rigidity, and keeps its own shape; as the ice melts, it loses its rigidity and becomes a fluid.

If we take a cubical block of ice, and, holding one face fixed, apply force to the opposite face tending to make it slide parallel to itself, and so to change the form of the block without altering its volume, we experience resistance because of the rigidity of the ice. When the ice has become water its particles can be made to slide over each other by the action of the very smallest force. The fluid has no rigidity, that is, no capacity of resisting such motion. It follows from this that the essential property of a fluid is that, when at rest, the pressure which it exerts on any surface with which it may be in contact, is at right angles to that surface; for, if not: suppose A B (fig. 3) to be the surface, and let the fluid exert pressure on it in direction C D, not at right angles to A B. The surface presses the fluid in direction D C.

Let D E be at right angles to the surface. We can resolve the pressure along D C into two components, one along D B, the other along D E. The former of these tends to make the fluid particles in contact with the surface slide over the next layer of fluid, and since the fluid has no rigidity, there is nothing to resist this tendency, and motion will ensue, which is contrary to the hypothesis that the fluid is at rest. Thus there can be no force exerted by the surface in the direction D B; the whole force is therefore along D E; hence, the whole action of the fluid on the surface is along E D, and this is the proposition we wanted to prove.

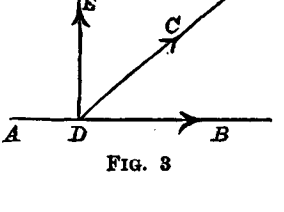

FIG. 3

Again, we can subdivide fluids into two main classes. Some fluids are almost incompressible; with them it requires an extremely great force to produce even a small diminution of volume. These fluids are called liquids. Imagine a cylinder filled with water closed by a tightly-fitting piston; by placing a great weight on the piston we could force it down somewhat. Thus it has been shown that if the piston contain 100 square centimetres in area, and the height of the cylinder be 10 centimetres, so that there is a litre of water in the vessel, then, if we place a weight of 100 kilogrammes on the piston, so that each square centimetre supports 1 kilogramme, the piston will be

depressed by about one two-thousandth of a centimetre. This is calculated on the supposition that the walls of the vessel do not give at all under this pressure; of course, in practice, they would yield appreciably, and allowance would have to be made for this. Such substances, then, as water, oil, alcohol, mercury, and many others, are nearly incompressible fluids, though, of course, their degrees of compressibility vary. They are classed together as liquids; they have no rigidity, but offer great resistance to compression.

The fluids in the second main subdivision are very easily and readily compressed, and are called gases. A gas is a fluid because it has no rigidity; it differs from a liquid because it readily expands and contracts under variations of pressure. Thus, if the cylinder in the experiment just described were filled with air or some other gas instead of with water, and the same weight were placed on the piston, it would sink through about 5 centimetres, or, in other words, the volume would be nearly halved. In practice this experiment would not be carried out exactly in this manner, for the friction between the side of the cylinder and the piston would produce error. This, however, can be avoided by a suitable modification of the apparatus. Thus air is about ten thousand times as compressible as water. But there is yet another distinction. Suppose the sides of the cylinder are continued some distance above the piston, and that the piston is raised, the level of the water will still remain at about 10 centi-

metres from the bottom. Above it there will be an almost empty space, containing merely a small quantity of water vapour, while the pressure on the surface of the water will be inappreciable. There will be a free surface to the water separating it from the space above. If, however, the cylinder be filled with air or some other gas, this will not be the case. The gas will rise with the piston, and fill the whole space below it almost uniformly. The pressure exerted by the gas on the sides of the cylinder and on the piston will be reduced; force will be required to hold the piston up. When the piston has been raised 10 centimetres, the volume of the gas will have been doubled; its pressure, we can show, will have been reduced to about half of its initial value.

Again, consider a substance like honey or treacle. We can pour it from one vessel to another; if we make a heap of it on a plate or dish, the heap is gradually flattened out till the dish is covered. It will not retain the shape to which we mould it for more than a brief time, but the rate at which it changes its shape is slow. It has no rigidity, and therefore must be classed as a fluid; yet it clearly differs greatly from water or alcohol, which, when poured out into a dish, cover the whole area of the dish almost immediately. With treacle a very small force is sufficient to change its shape, provided that force act for a sufficient time. Such fluids are called viscous. A perfect fluid is one which yields instantaneously to a shearing stress—that

is, a stress tending to make its particles slide over each other, and so change its form without altering its volume; a viscous fluid yields in the end to shearing stress, however small the stress may be, but time must be allowed for the action to take place. In reality we know no such thing in nature as a perfect fluid; water, alcohol, &c., are slightly viscous, but, compared with treacle, honey, or glycerine, they may be treated as practically perfect fluids.

While there are these three well-marked groups of solids, liquids, and gases, including in the latter group vapours, as to the properties of which we shall have more to say shortly, it is not always easy to say to which group a given body belongs, the properties of the one merging to some extent into those of the other. Thus, consider a stiff jelly made by dissolving glue or gelatine in hot water, and allowing it to cool. Such a substance is a solid, it has rigidity, you can mould it into any shape you will, and it will retain the form given to it. But now add more water to the mixture; it begins to lose some of its solid properties, and after a time you get a sticky liquid like gum. It is impossible to say exactly at what point the substance changed from solid to liquid. The fluid certainly is a viscous one at first; continue, then, the process, and add still more water, the viscosity is reduced until you would class the substance as a non-viscous fluid. But at what stage in the proceedings you can fairly say the fluid has ceased to be viscous is an open question.

Once more. Beeswax or paraffin are solids. They will slightly resist a shearing stress, but their limits of elasticity are very small. The application of quite a small force is sufficient to mould them to a new form; they are very plastic. Many solids have this property to some extent. If the stress applied to them exceeds a certain limit, they give way and take a new form, in which the stress is reduced. A further application of force ruptures or breaks them, but there is a considerable range between the force required to strain them beyond the limits of elastic recovery and that which will rupture them. The terms tenacious, ductile, and malleable, as applied to metals, express similar properties.

Thus, copper is very ductile; it can be drawn out into wire by pulling it repeatedly through holes in a steel or agate plate, and by this process its diameter may be reduced many times without rupture or breach of continuity. Copper is also malleable. A lump of it can be hammered out into a thin sheet, or bent by the action of pressure applied through a mallet into the form of a bowl or mould. Gold is very malleable, and can be beaten out into extremely thin sheets of gold leaf without impairing its continuity. Gold can be drawn into wire, but not so readily as copper. Lead, again, is a malleable substance; but it is not so tenacious as copper. A malleable substance yields gradually to the effect of pressure, contracting in the direction of the pressure, and expanding in directions at right angles to

it. A tenacious substance will stand considerable tension without breaking, extending gradually in the direction of the tension, and contracting at right angles to it; a plastic substance is one in which the property of malleability is in excess, so that the main elastic feature of the body is the readiness with which it can be moulded to any desired form. Copper is ductile because it is both plastic and tenacious. There is, however, a distinction to be observed between a plastic and a ductile body; both are rigid up to a certain limit. In a plastic body, however, this limit is fixed; when subjected to a distorting stress exceeding the limit the body gives way. The amount by which the stress can exceed this limit without producing rupture depends on the tenacity of the body. In a ductile body, however, the limit is not fixed. To produce continuous yielding it is necessary continually to increase the distorting stress. As the body yields, its capacity to resist distorting stress increases, so that after a certain amount of yielding, if the distorting stress remains constant, the resistance of the body will become sufficient to balance the stress, and a new equilibrium position is found.

As a contrast to plastic, tenacious, or ductile bodies, we have brittle substances, such as glass or crystals, chilled steel, or china. These give way almost as soon as the limits of elasticity are reached. Strain a piece of glass up to a certain limit, it will return to its original form when the strain is removed;

attempt to strain it even but a little beyond that limit, and the glass fractures at once; it is highly brittle. Some substances are brittle when stress is suddenly applied to them; but plastic or even viscous if the same stress is applied more gradually. Pitch is an example of these. A sudden blow will fracture a lump of pitch instantly, while gentle pressure applied continuously for some time will mould it into almost any required form. Pitch is brittle for a sudden blow, viscous for slow, continued action.

The properties of plasticity and viscosity merge into each other, and it is sometimes difficult to draw the line between a soft solid and a viscous fluid. Mere difference in hardness is not sufficient. Wax is softer than pitch, and can easily be cut by a knife, while pitch is fractured at once when the attempt is made, and yet pitch is strictly a viscous liquid, for if a lump of pitch be placed on a table and left merely under the action of so small a force as its own weight, it will flatten out like treacle, only at a much slower rate, and lose its form, or again, if pitch be placed in a funnel and left, it will in time flow down through the funnel in a slow continuous stream. Any small force then, if it act for long enough on pitch, will change its form; thus it is a liquid, though a very viscous one. To mould a lump of wax the force applied must exceed a certain limit; the weight of the wax is insufficient to change its form; it is thus a solid. Of course this is a very extreme case; it is merely given to illustrate the way in which

solid and fluid properties are connected together, and the criterion to be used to distinguish them. For small rapidly-acting forces pitch is a rigid solid; tuning-forks have been made of it which, when struck, have for a time given an audible note; they collapse, however, under their own weight very shortly.

Many substances can exist under varying conditions in all the three states above mentioned; the temperature is generally the condition which determines the state a substance takes. Thus, at low temperatures water-stuff takes the form of ice, which is solid, and has some rigidity. It is, perhaps, still an open question how far ice is plastic or viscous; it certainly does mould itself, in time under pressure, to varying forms; but that may be due to the phenomenon of regelation, which could produce the effect. The recent most interesting experiments of Messrs. McConnel and Kidd have shown, however, that glacier ice is a conglomerate of crystals, and that while each crystal is rigid for stresses applied in certain directions it will gradually yield like a viscous body to the continued action of stresses applied in other directions.

When heat is applied to very cold ice its temperature rises, but at a certain temperature, depending very slightly on the pressure, it changes to the liquid water; above that temperature it cannot exist as ice; below it, except under certain quite abnormal conditions, it cannot have the form of water. When the change is complete the addition of more heat raises the

temperature again, and the rise goes on continuously till another temperature is reached, depending to a much greater extent on the pressure, at which the water substance becomes steam, and as such has in the main the properties of a gas; it will expand and fill the space above the water like a gas. It is, however, in one very important particular different from a gas. Let us consider this more closely. Suppose we have a cylinder filled with air, and that we compress the air by pressing down a piston, keeping the temperature constant; the volume of the air is decreased and its pressure increased. Now repeat the same experiment with steam instead of with air; the result will depend on the pressure in the cylinder originally, but probably for a time the pressure will increase as the volume decreases. This law, however, will not hold for long. As the piston is still further depressed, some of the steam will be condensed into the liquid form, and the pressure of the remainder will remain the same; the effect of pushing the piston still further down will be to condense more and more steam, until at last we have only liquid beneath the piston. Steam is a vapour as distinguished from a gas, and the first rough distinction we draw between them is that a vapour can, at ordinary temperatures, be condensed to the liquid form by the action of pressure alone, while a gas cannot; the words 'at ordinary temperatures' will be found in the sequel to be of importance.

CHAPTER VI

TERMS APPLIED TO PROPERTIES OF MATTER—ELASTICITY, RIGIDITY, COMPRESSIBILITY

LET us now see if we can define and measure rather more exactly some of the terms used and explained in the last chapter as applied to matter. Some of these terms express qualities which may be capable of comparison but not of exact determination. Thus, hardness is a relative term, and we have no absolute scale to measure it by; the statement that one substance is twice as hard as another conveys no meaning unless we say how hardness is to be measured. We may establish an arbitrary scale, as is done by mineralogists who have taken certain standards of hardness, viz., the hardness of each of the minerals in a certain series. If a given mineral scratches one member of this series, but is itself scratched by the next, then its hardness lies between those of the two minerals in question. But this does not give us a measure of the hardness, in the same way as we can measure, say, the density of the body. For the density of a homogeneous body is the mass or quantity of matter in the unit of volume, and is determined by

finding the number of units of mass and the number of units of volume in the whole body and dividing the former of these two by the latter; it is thus expressed as so many grammes or pounds to the unit of volume and becomes completely determinate. Now, many of the properties we have been dealing with, such as rigidity, compressibility, viscosity, and others, are mechanical in their nature, and can be expressed in terms of the fundamental units. Thus, rigidity measures the resistance a body offers to change of shape; it is therefore properly determined by the stress required to produce some definite change of shape. Resistance to compression, again, will be measured by the stress required to produce some specified compression, and so on. Let us try to analyse some of these quantities a little further.

We will deal only with isotropic bodies; that is, with bodies which have the same properties in all directions round a point.

Take a cubical block of an elastic solid and let one face of it be held fixed. Suppose forces to be applied uniformly to the opposite face in such a way as to make it slide parallel to itself a small distance, the planes intermediate between the two will also slide forwards though to a less extent, and the distance any plane moves through will get less and less as we approach the fixed plane. Thus, suppose the fixed plane to be at right angles to the paper, and let it cut the paper in A B (fig. 4), and let the plane which is made to slide forwards cut the paper in C D. Suppose that by the motion it is

brought to the position C′ D′, the line A C will become A C′ and the line B D, B D′. The body is said to experience a shearing strain, and the amount of the shear is measured by the ratio of C C′ to C A. Moreover, if E be any point on A C, and if E E′, F F′, drawn parallel to A B, cut A C′ and B D′ in E′, F′ respectively, then by the motion E is brought to E′ and F to F′. We have also E E′/*E A = C C′/C A = the shearing strain. The force acting on the face C D to produce this will be a uniform traction, parallel to the line C D; the amount of this traction per unit of area of the face is called the shearing stress, and it is found by experiment that when the strain is sufficiently small the ratio of the shearing stress to the shearing strain is constant. This statement, that the ratio of a stress to the strain it produces is constant, constitutes the general expression of Hooke's law. Hence, if we denote by T the shearing stress and by a the corresponding strain, then T/a is always the same for the same substance under given conditions. Let us denote this constant ratio by n, then we have $T = n\,a$. Let us now consider the question what stress will produce unit shear, in this case $a = 1$ and therefore $T = n$; so n is the stress required to produce unit shearing

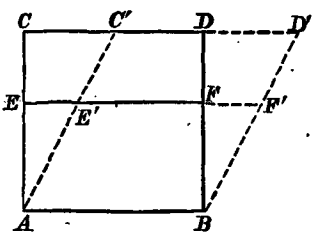

FIG. 4

* The sign / is now used as an abbreviation for 'divided by.' Thus E E′/E A denotes the ratio of E E′ to E A.

strain; n, therefore, is a measure of the rigidity of the body and is called the modulus of rigidity. Of course we could not really produce unit shear in most bodies, the body would rupture long before it was strained to this amount, and the stress before rupture would probably have ceased to be proportional to the strain. We may define the simple rigidity, then, as the ratio of the shearing stress to the shearing strain when the strain is small; if the laws of elastic solids held over so large a range of strain, the simple rigidity would be measured by the stress required to produce unit shear.

The displacements involved in a simple shear may be illustrated by taking a thick and somewhat loosely bound book, and while holding the one cover against the table, pushing the upper cover parallel to the table; the leaves thus all slide parallel to themselves through spaces which are proportional to their distances from the lower cover. The book is sheared. The force applied per unit of area to the cover is the shearing stress; the ratio of the distance moved by the cover to the thickness of the book is the strain.

Or again, take a cube composed of a large number of square cards placed one on the top of the other like the leaves of a book. Place some elastic bands round it to hold the cards together; in this position it represents the unstrained cube of solid; put one set of edges of the cards in contact with the table and press obliquely on the top surface; the cards will slide over each other, and on turning the block so that the lowest card again lies on

the table it will be found that the block is no longer a cube, having six square faces and all its angles right angles, but that while the top and bottom faces remain square, the others have become parallelograms. The cards now represent the solid when strained by a simple shear. This method of considering the question brings out one fundamental property of a shear. It is clear that the volume occupied by the cards or by the leaves of the book is the same in each of the two positions, and so we infer that the volume of a body is not altered by a shearing strain. This follows from fig. 4, for since the triangle C′ C A = triangle D′ D B, we see that the area C A B D = area C′ A B D′, and the same is true for any section of the body by a plane parallel to the paper. Hence, a block of the substance with its faces passing through C A, A B, B D, and D C respectively, and at right angles to the paper, becomes strained into one of equal volume but with its faces through C′ A, A B, B D′ and D′ C′ and still at right angles to the paper.

We have thus investigated the meaning of a simple shear, and have shown that in a body of rigidity n, the shearing stress and strain are connected by the equation

$$T = n\,a.$$

If the stress is too great for the body to stand, it yields to it and separates into two portions, the one part sliding as it were over the other parallel to the plane of shear; thus, when a hole is punched in a sheet of lead, a very intense shearing stress is applied over a

limited surface, the metal gives way and a small cylindrical bit is forced out by the punch. The same sort of action takes place when a sheet of lead or copper is cut by a pair of shears; the lower blade of the shears supports one half of the sheet, the upper blade presses the other part down, and the two parts are sheared asunder.

In order to strain an elastic solid it is necessary to do work; assuming no energy is lost as heat, that is, that the temperature remains constant all the time, the whole of this work is stored up as the potential energy of strain in the solid and can be recovered by allowing the solid to regain its equilibrium condition. A compressed spring or a bent bow are examples of strained solids, and have energy which may be used, in the one case to drive the mechanism of a clock, in the other to propel an arrow.

Thus it is a natural question to ask, How much energy is stored in a body which is subjected to a simple shear?

If the stress did not depend on the strain, the answer would be a very simple one. Let us suppose the body is strained by a stress T applied per unit of area, and that we consider the work done in increasing the strain by a very small amount, so small that we may consider T as not appreciably varying during this change. Let l be the length of a side of the original unstrained cube which we are dealing with. The area of a face is thus l^2 and the force which strains it is $\mathrm{T}\,l^2$. Take a section of the cube, as

in fig. 5, and, in consequence of the additional small strain, let C′ be brought to P and D′ to Q. Then C′ P = D′ Q = x, say; and by this strain the point of application of the force T l^2 has moved a distance x. Thus the work done is T $l^2 x$. Then the additional strain = C′ P/C A = $x/l = a'$, say. Hence the increase of energy is T $l^3 x/l$ or T $l^3 a'$ and the increase of energy in a unit of volume is T a'. It must be remembered that a' is supposed so small that T may be treated as constant and equal to its average value during that strain.

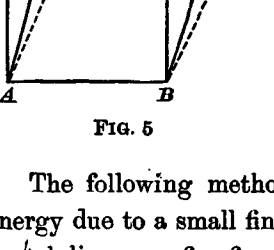

FIG. 5

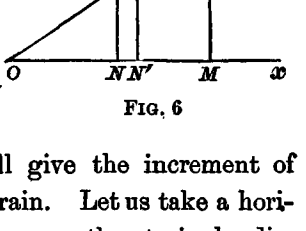

FIG. 6

The following method will give the increment of energy due to a small finite strain. Let us take a horizontal line, O N, fig. 6, and measure the strain by distances, O N, &c., along this line and the corresponding stress by vertical lines P N, &c. These are called ordinates. Let O N denote the strain a, and N N′ the additional strain a'. If P N = T, then the increment of energy per unit volume is represented by the parallelogram P N N′, and the whole energy in any finite small strain will be the sum of such parallelograms. Thus, if we

draw a curve O P Q, such that the ordinates of this curve represent the stresses, the area of the curve will represent the energy, but since by hypothesis the strain is so small that the stress is proportional to it, the curve is a straight line, O P Q say, and if Q M be perpendicular to O N and represent the final value of the stress, O M being the final strain, then the energy = Area O Q M = $\frac{1}{2}$ O M . Q M = $\frac{1}{2}$ $a \times n\, a = \frac{1}{2}\, n\, a^2$, if a is now the final strain. Thus the energy is one half of the final stress multiplied by the final strain, or is the average stress multiplied by the final strain. For since the stress increases uniformly with the strain, the average stress is half of the final stress.

The energy can be expressed also in terms of the final stress, for if this be T we have $T = n\, a$.

Thus the energy per unit volume is $\frac{1}{2}\, T^2/n$.

These results give us another meaning for n, for since the energy is $\frac{1}{2}\, n\, a^2$ we see that if $a = 1$ or the body suffer unit shear, then the energy = $\frac{1}{2}\, n$, that is, the rigidity is measured by twice the energy stored per unit volume, when the body experiences unit shear.

The foregoing is, of course, only true when the strain is a simple shear, that is to say, when the body undergoes only change of form and not change of volume. Let us now take the case of a body undergoing change of volume only and no change of form; any cube in the body remains a cube, only the length of its edges is changed. Let us take a cube having sides of length l; suppose that three of the faces which meet at one of its

angles are fixed and that we consider the effect of applying equal pressures, P, per unit of area, to each of the other three. Each edge will become smaller by a length x, suppose. The ratio x/l, which is the decrease in length per unit of length, is called the linear contraction.

The volume of the unstrained cube is l^3, that of the strained cube is $(l-x)^3$, and this is equal to $l^3 - 3\,l^2\,x + 3\,l\,x^2 - x^3$; but we suppose x is very small, so that x^2 and x^3 are much smaller than x; hence we may put for the new volume $l^3 - 3\,l^2\,x$. Thus the decrement in volume is $3\,l^2\,x$; the cubical contraction is the ratio of the decrease in volume to the original volume, and this is $3\,l^2\,x/l^3$ or $3\,x/l$.

Thus, in this case, the cubical contraction is three times the linear contraction. Let us denote the cubical contraction by δ; then, within the limits with which we are concerned, experiment shows that the pressure required to produce a cubical contraction δ is proportional to δ, and we have therefore P/δ = a constant = k, say. Thus $P = k\,\delta$; k is the pressure required to produce unit contraction, or the tension required to produce unit dilatation; it therefore measures the resistance to compression.

Again, let us find the work done in producing this contraction δ; this will give us the energy stored in the strained solid; the work done on one face in increasing the displacement x by an amount x', so small that for this small change we may treat P as constant,

is $P l^2 x'$, and this is equal to $P l^3 e'$ if e' is the corresponding change in the contraction e. Thus the work done per unit of volume is $P e'$ and the work done per unit of volume in straining the body from its initial state to its final contraction e is $\frac{1}{2} P e$. Taking account of each face, we get as the total work done $\frac{3}{2} P e$ or $\frac{1}{2} P \delta$, and since $P = k\delta$ the total work done in producing a dilatation δ without any change of shape is $\frac{1}{2} k \delta^2$; we might therefore state that k is measured by twice the energy per unit of volume when the body experiences unit dilatation or contraction without change of shape. A more general case of strain is when the body experiences both change of volume and change of form; this may be conceived as made up of three elongations or contractions e, f, g, of unequal amount, parallel to the edges of the cube, combined with three shears α, β, γ, in which the sliding motion takes place parallel to each pair of faces in turn; the first set of strains change the cube into a rectangular parallelopipedon, while after the shears the parallelopipedon ceases to be rectangular. We may show in this case that the dilatation is $(e+f+g)$ while the energy is $\frac{1}{2}[k(e+f+g)^2 + \frac{2n}{3}\{(e-f)^2 + (f-g)^2 + (g-e)^2\} + n(\alpha^2 + \beta^2 + \gamma^2)]$, but the proof would be rather complicated.

From this expression for the energy we can calculate the stresses which will be exerted across any given surface in the solid; we can also find the stresses which

must be applied to the surface to produce a given state of strain; or again, we can obtain equations to give the strains, and hence the displacements at each point of the solid produced by the application of known forces to its surface. In general the solution of the equations is not easy, and has only been completely obtained in some special cases.

The two quantities n and k, as defined above, constitute the two principal moduluses of elasticity for an isotropic solid; there is, however, another quantity connected with the above on which some important physical properties of the solid depend, more directly than they do on the two principal moduluses, and which is capable of comparatively easy determination. Suppose we have a long bar of elastic material, and apply to it a tension in the direction of its length; the bar will then expand in the direction of its length and contract in directions at right angles to the length; the amount of this expansion for a given tension depends on the value of *Young's Modulus* for the material. This quantity is defined to be the ratio of the tension per unit of area to the extension per unit of length produced by it, and is found to be constant if the tension be not too great. Thus, if a force of T dynes be applied, and if the area of the cross section of the bar be σ square centimetres, the tension on each unit of area is T/σ; also if the length of the bar before stretching is l centimetres, and after stretching l' centimetres, the extension is $l'-l$ and the extension per unit of

length is $(l'-l)/l$. Hence Young's Modulus for the material of the bar is $\mathrm{T}\, l/\sigma\,(l'-l)$. This assumes that the extension is proportional to the tension producing it, *i.e.* that within the limits considered Hooke's law holds. The law mainly stated with reference to this case was given by him as an anagram in 1676, the anagram being *ceiiinossstivu*; the key to the anagram followed in 1678, in the words *ut tensio sic vis*—as is the extension so is the force. Our reasons for believing the law are of course based on experiments made on elastic bodies. We can take a rod or wire and suspend it vertically, hanging a scale pan from it; we then measure its length, and on putting weights into the scale pan, we find that the wire extends. On measuring by suitable apparatus its extension we find that, if the weights do not exceed certain limits, the extension produced is proportional to the weights placed in the pan. Now if we suppose the rod or wire to be isotropic[1] we can show from the general theory of elasticity that the ratio of a simple tension in one direction to the elongation it produces is $9nk/(3k+n)$. This, then, is the value of Young's Modulus for an isotropic medium, while the ratio of the contraction in directions at right angles to the length to the elongation in the direction of the tension is given by $(3k-2n)/2(3k+n)$.

This quantity is very often called Poisson's ratio.

[1] A wire is almost certainly not isotropic, the process of 'drawing' gives it different properties along its axis to those which it possesses in directions at right angles to the axis.

It is a consequence of a certain mathematical theory of elasticity, developed in France by Poisson and Navier, that this ratio should be $\frac{1}{4}$; for according to them there is a constant ratio between n and k, such that for all substances $5n=3k$. Stokes pointed out that there were many substances for which this result is certainly false. Jellies and india-rubber have very different rigidities, and are all nearly incompressible, so that for them k is much greater than $\frac{5}{3}n$, and in fact since the volume remains nearly constant if a column of india-rubber be pulled out, it is found that the contraction at right angles to the length is approximately half the elongation, so that Poisson's ratio is $\frac{1}{2}$ not $\frac{1}{4}$. Stokes also gave reasons for thinking that in metals k was probably greater than $\frac{5}{3}n$, and the experiments which have since been made on the subject have shown that in general Poisson's ratio is greater than $\frac{1}{4}$, thus bearing out Stokes's reasoning. Its value for various substances is approximately as follows:

Glass	·333
Brass	·387
Iron	·294
Copper	·226 to ·441

Cork, on the other hand, is much more rigid than the theory would require, for it can very readily be greatly compressed or extended in length without appreciably altering its lateral dimensions.

We are therefore justified in treating n and k

as two independent constants on which the purely elastic properties of a body depend, though, as we have said, it is the quantity $9nk/(3k+n)$, or Young's Modulus, which is important for many practical purposes.

Thus the bending of a beam depends of course on its length, weight, and cross section, but it also depends directly on Young's Modulus for the material of which it is made. For consider a beam fixed at one end so that if unstrained it would rest in a horizontal direction, but free at the other, and therefore strained and deflected from that position partly by its own weight and partly by the action of a weight hung from the free end; the fibres on the upper side of the beam are clearly stretched by the bending, those on the under side are compressed. Now, if we have two beams of different materials, but otherwise exactly alike, and loaded with equal weights, and take two transverse sections, one in each beam, at the same distances from the fixed end, the stresses tending to produce elongation or compression are the same at corresponding points of the two beams, but the compression or elongation produced will be proportional to the stress and inversely proportional to Young's Modulus; the curvature of the beam at each point will depend on the compression or elongation, that is to say on Young's Modulus, and thus the whole deflection, if not too large for Hooke's law to be true, will be inversely proportional to Young's Modulus.

A figure may perhaps make the argument rather

clearer. Let A B D C (fig. 7) represent the beam when bent. Let $a\,c$ and $b\,d$ be two transverse sections, which, in the unbent state, were vertical; since the uppermost fibres, such as those along A B, are stretched while the under ones, such as C D, are compressed, there must be some layer which remains of its natural length.

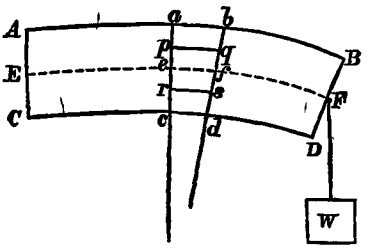

FIG. 7

Let this layer be E $e\,f$ F; now take a fibre, $p\,q$, above $e\,f$; its original length was $e\,f$, thus its increase in length is $p\,q - e\,f$, and its elongation, or the ratio of this to the original length, is $(p\,q - e\,f)/e\,f$; hence, if M stands for Young's Modulus, the tension along $p\,q$ per unit of area of the cross section at p is $\mathrm{M}\,(p\,q - e\,f)/e\,f$. At the same time there is a pressure at r, if $r\,s$ is a fibre below the unstretched one, given by $\mathrm{M}\,(e\,f - r\,s)/e\,f$. The tension tends to shorten $p\,q$, the pressure to elongate $r\,s$, the result of the combined actions for all the elements of the section $a\,c$ is to tend to raise the end B D of the beam; the effect of the weight W, which we suppose to be suspended from F, and of the weight of the portion a B D c of the beam is to lower the end B D; the one effect balances the other, and from this condition we can determine the form assumed by the beam. Now, if we have two beams of the same size and shape, but of

different materials, and suppose them deflected by the same weight, the tensions and pressures at corresponding points of a section $a'c'$ of the second beam must be the same as those just investigated, and hence we shall have, if M′ be Young's Modulus for the second beam,

$$M'(p'q'-e'f') = M(pq-ef);$$

but the bending of the small portion between the planes ac and bd depends on the value of $pq-ef$, and hence we get, if y represent the total deflection of the one beam, y' of the other, that $My = M'y'$, or the deflections are inversely proportional to the values of Young's Modulus.

The complete investigation shows that if W be the weight deflecting the beam, and if this be great compared with the weight of the beam, so that the effect of the latter need not be considered, if l be the length of the beam, b its horizontal breadth, and d the vertical depth of its section, which we suppose to be rectangular, then

$$My = \frac{4Wl^3}{bd^3}.$$

By observations on the deflection combined with measurements of the length, breadth, and thickness of the beam the value of Young's Modulus can be found.

The relation between the tension required to elongate a bar of cross section σ square centimetres the extension produced and Young's Modulus is, we have seen, the following: $M = \frac{Tl}{\sigma(l'-l)}$.

Suppose, then, we have a bar one square cm. in cross section and then inquire what force is necessary to double its length; assuming of course Hooke's law to hold for such an extension (this assumption would not be true in most natural substances), then $\sigma = 1$ $l' = 2$ l. Thus $M = T$. Hence Young's Modulus for a given material is the tension required to double the length of a bar of unit cross section of that material.

The greatest amount of longitudinal stress which a given body such as a wire can bear will depend on its cross section and on the tenacity of the material; the tenacity is represented by the greatest stress which a wire or bar of unit cross section can bear without rupture. Thus, if a bar of cross section σ square centimetres break when a tension of T dynes is applied to it, the tenacity is T/σ dynes per square centimetre. The tenacity is determined by suspending the wire or bar and loading it till it just breaks.

Let us now consider briefly some methods of finding the values of n and k, the rigidity and the resistance to compression. One relation between these quantities is given when Young's Modulus is known, while Poisson's ratio gives another, and from these two the values of n and k can be calculated. Accordingly experiments have been made with the view of finding Poisson's ratio directly; this might be done theoretically by taking a uniform bar of the material and measuring its length and diameter when free, the bar would then be subjected to great longitudinal stress and the length and diameter

again measured. From these data the ratio of the lateral contraction to the longitudinal extension could be found. In practice, however, the change in the diameter of the bar is much too small to be thus measured with accuracy, and the difficulties of obtaining good results are very great; it is better, therefore, to have recourse to other methods for finding n and k. Now, if we take a long column of circular cross section and, holding one end fixed, twist the other end about the axis of the bar, we do not alter the volume of the bar; the strain produced is a simple shear. The stress thus introduced into the bar will depend only on its length and cross section and on the rigidity of the substance, and moreover it will clearly be proportional to the rigidity; thus we can compare two rigidities by finding the forces necessary to twist two bars of the same dimensions through the same angle. To measure the rigidity of a given substance we must calculate the stress produced by twisting a bar of that substance through a given angle. This stress in any cross section of the bar takes the form of a couple tending to untwist the bar, and if a is the radius of the bar, l its length, and θ the angle which the one end of the bar is twisted through relatively to the other, we can show that this couple is given by $\frac{n \pi a^4}{2 l} \theta$.

Using, then, this result, we have to equate this couple to the known couple employed to twist the bar, and from the equation we can deduce the value of n.

This is a statical method of finding n, for we equate

the torsional effect to a statical couple. We might equally employ a dynamical method as follows. Suppose a mass of known form—a sphere or cylinder, say—is suspended by the wire. Turn the mass through a small angle, thus twisting the wire about its axis, and then let it go. The system will continue to oscillate backwards and forwards round the vertical axis, and the motion will be more rapid the greater the force due to the torsion. When the wire was twisted it had potential energy; this potential energy is transformed into the kinetic energy of the mass, the transformation being complete when the wire is completely untwisted, but the momentum acquired by the mass carries it through this position, twisting up the wire in the other direction and transforming the kinetic energy into potential. If we could omit from consideration the energy dissipated as heat in the wire, and by the friction with the air, the system would go on oscillating continually. We can show as follows that the time of an oscillation is inversely proportional to the square root of the rigidity: consider two wires of the same length and thickness, but of different rigidities, let them carry equal masses, and suppose the two twisted through the same angle; their potential energies will be proportional to their rigidities. Now consider what has happened when each wire has untwisted through the same angle; each has lost potential energy and gained kinetic, the loss of the one form of energy being equal to the gain of the other form. Also, since the

angle twisted through is the same for the two, the loss in each case will be proportional to the rigidity.

Thus the kinetic energies of the two masses, when the angle of twist is the same for each, are proportional to the rigidities of the wires; hence the velocities at such corresponding points are proportional to the square roots of the rigidities, and the times of describing equal small arcs near these corresponding points will be to each other in the inverse ratio of the square roots of the rigidities. Hence the whole time of an oscillation of the one will be to the whole time of an oscillation of the other inversely as the square root of the rigidity of the first to the square root of the rigidity of the second.

By considering the complete problem, it can be shown that if T is the time of the complete oscillation, K a quantity depending on the form and material of the suspended mass, and called the moment of inertia of that mass, then $T = 2\pi\sqrt{\left\{\frac{2Kl}{n\pi a^4}\right\}}$, l being the length of the wire, and a its radius.

Hence $n = \frac{8\pi K l}{a^4 T^2}$.

Now all the quantities on the right hand side of this equation can be determined, and thus a value can be found for n.

If now we know the value of n and M for the same substance, we can deduce the value of k as follows. We have seen that $M = \frac{9nk}{(3k + n)}$; from this we find

$k = \frac{n\,\text{M}}{3(3n - \text{M})}$, which gives us k, but the method does not lead to very accurate results, since k is for many substances large compared with n, and thus $3n - \text{M}$ is small, so that a small error in the value of n or M is important. Thus for drawn brass in certain units we have M = 1096, while n = 373, hence $3n - \text{M} = 1119 - 1096 = 23$; but if we suppose that n is in error by less than 1 per cent., so that its true value is 370, then, $3n - \text{M}$ becomes 14, and the error in k will clearly be much greater than that in n. Another method has, therefore, been adopted in some few cases. A portion of the substance to be examined is enclosed in a vessel, in which it can be subjected to great pressure by pumping in water, or some other liquid; portions of the vessel are of thick plate glass, strong enough to stand the pressure, and the distance between two marks on the substance, visible through the glass, can be determined by the aid of microscopes. On compressing the water, the distance between the marks is decreased; from this decrease the linear contraction due to a uniform pressure all over the surface is obtained, the cubical contraction is, we know, three times the linear, and the ratio of the applied pressure to the cubical contraction gives the value of k.

Measurements have been made of one or two other quantities related to the elastic properties of bodies. Of these, perhaps, the most important are the practical limit of elastic elongation and the resilience. By the

practical limit of elastic elongation is meant the maximum amount of elongation which a wire or bar will stand and completely recover from when the stress is removed.

By the resilience is denoted the amount of energy in the strained body, per unit of volume, and it is generally used for the maximum amount of energy due to some particular type of strain when the strain has reached its maximum value consistent with complete elastic recovery.

The strength of a material for a given type of strain is measured by the stress which will produce the limiting strain of that type. Thus the elastic strength corresponds to the limit of perfect elasticity, the ultimate strength to rupture.

The tenacity is the ultimate strength for elongation. A simple longitudinal pull greater than this will produce rupture.

The following Table, taken in the main from Sir William Thomson's article in the 'Encyclopædia Britannica,' and quoted by Ibbetson, will give the values of these quantities for a few materials.

Material	ρ	M	n	k	E	V	T
Flint glass .	2·942	615×10^6	243×10^6	437×10^6	—	—	—
Steel piano wire. . .	7·727	2049×10^6	—	—	·0115	135995×10^4	2362×10^4
Iron, wrought	7·790	2040×10^6	784×10^6	1484×10^6	·00224	5120×10^4	457×10^4
Copper wire.	8·9	1185×10^6	445×10^6	1172×10^6	·0036	7480×10^4	422×10^4
Brass wire .	—	1001×10^6	410×10^6	597×10^6	·00344	5905×10^4	343×10^4

In the Table ρ is the density in grammes to the cubic centimetre. E is the practical limit of elastic elongation. The value of the moduluses M, n, and k are given in grammes weight to the square centimetre; to reduce them to dynes to the square centimetre they require to be multiplied by 981, the value of g. The tenacity, T, is given in the same units, while V, the resilience for longitudinal extensions, is in gramme centimetres per cubic centimetre, and also requires multiplying by g to reduce it to ergs per cubic centimetre.

In the preceding chapter a distinction has been drawn between ductile and plastic bodies. Let us now consider this a little more closely. Both have complete elasticity of volume; if compressed by uniform pressure applied to each point of the surface, they contract, but recover their volume when the pressure is removed. Both have, up to a certain limit, elasticity of shape. They will recover completely from the action of a shearing stress provided that stress be less than a certain amount. We will denote this limiting shearing stress by S and call it the solidity of the body; if the shearing stress exceeds S the plastic body has no capacity to resist the excess of stress above S, and yields until, by the yielding, the stress has again been reduced to the value S. When the stress is removed, the distortion produced by this yielding remains as permanent set in the body; the resilience is the same as if the body had only been strained to its natural limits of elasticity and that amount of strain alone is recoverable,

the values of n and k being unaffected by the strain. In a ductile body the quantity S, which measures the resistance to flow, is not a constant but increases with the flow, so that if a shearing stress greater than S be applied the body flows, and the flow continues until the resistance to flow has risen to such an amount that it can balance the distorting stress. Let this new value of the resistance be S′, then on the next application of a shearing stress it will be found that the elastic limit has been raised; flow will not begin when the shearing stress reaches the value S as previously, but the stress can be extended to the value S′ without causing it.

By this process the metal is 'hardened,' and 'hardness' is used to indicate the resistance to flow of ductile metals, solidity being applied to plastic bodies. The hardness of a ductile body depends on its treatment and history, the solidity of a plastic body is a constant. The maximum value of the hardness coincides with the ultimate strength of the material under shearing stress. When the substance is as hard as possible it is brittle, and ruptures immediately it is strained beyond its elastic limits. It may be softened again by heating under proper conditions, and when so softened will have its ductility restored.

Set affects the elastic symmetry of a wire, so that a wire or bar which has been subjected to it is not completely isotropic. As an example of the behaviour of a ductile metal under elongation, let us take a piece of soft copper wire, suspended in a vertical position and carry-

ing a scale pan, into which varying loads can be put. It will be found that probably at first, if it be loaded with a given weight, the wire extends, but does not completely recover itself when the weight is removed. If, however, the same weight, w, be applied and removed several times the residual set disappears, the wire reaches its state of ease, as it has been called by Professor K. Pearson. When this state has been reached, the extension is found always to be proportional to the load for weights less than w. By increasing w and repeating the process, Hooke's law may be shown to hold over considerable limits, and within those limits the wire behaves as a perfectly elastic body; if, however, the limit be exceeded, the wire extends suddenly by an appreciable amount, and on removing the load it is found to be permanently lengthened; the shearing stress applied has been greater than the natural hardness of the body, and it has yielded; the process of flow has increased the hardness until it balances the shearing stress; flow has ceased and the wire is now an elastic solid with its limits of elasticity considerably increased. Moreover, it is found that by this process the value of Young's Modulus is not appreciably changed, though the bar has probably ceased to be elastically isotropic.

This process can be repeated, and the wire or bar considerably extended, until at last the load exceeds the limits of tenacity, and the wire is ruptured. In the process hardening has gone on to a considerable extent, and the hardness has reached its limiting amount.

An account of the results attending the extension of an iron bar 10 ins. long and $\frac{3}{4}$ in. in diameter is given in 'Nature,' vols. xxxi., xxxii., by Professor A. B. Kennedy. These are carefully discussed by Mr. W. J. Ibbetson in his 'Mathematical Theory of Elasticity' (Appendix IV., on the Elastic Properties of Natural Materials), and much of the above is taken from his discussion.

CHAPTER VII

PROPERTIES OF FLUIDS—FLUID PRESSURE

A SOLID, we have seen, is distinguished by the property of rigidity; a plastic solid is one which can resist shearing stress so long as the stress is less than a certain limit, S, which we call the solidity. When S is vanishingly small, the substance loses its solid properties and becomes a fluid. A fluid can offer no permanent resistance to a shearing stress, though it may take time for the stress to produce its effect. Thus, as we have seen, the force exerted by a fluid on any surface with which it is in contact is at right angles to that surface.

Suppose, now, that we have a small plane area immersed in a fluid, the fluid presses upon both sides of the area, and if we could suppose the fluid removed from one side of the area without disturbing the equilibrium of the rest, we should have to apply a force to balance the fluid pressure on the other side. Let this force be P dynes, and suppose the area to contain σ square centimetres, P is the whole force the fluid exerts on the area; P/σ is the average force exerted

on each unit of the area. This quantity, P/σ, is called the average pressure *at* each point of the area. If we suppose the area to be extremely small, then the pressure on each little equal element of it will be the same, and P/σ is the pressure at the point at which the small area is placed. Thus the pressure *at* a point in a fluid is the limit of the ratio of the force on a small area immersed in the fluid and containing the point to the area when the area is very small, or, more simply, it is the force on a unit of area immersed in the fluid and containing the point, the force being supposed to be uniformly distributed over the area.

We will now show that we may apply the laws of statics to the forces which act on any portion of a fluid in equilibrium. For suppose we are considering any mass of fluid, and let P, Q be two contiguous particles in it; P exerts some action on Q, and Q exerts in return equal and opposite action on P. Hence, in considering the equilibrium of the mass of fluid, we may omit the action exerted by P on Q because it is balanced by the reaction Q exerts on P. The same is true for all such pairs of particles contained in the volume considered. The only forces we have to deal with, then, are those which are exerted from without on matter within this volume. Such will be the pressure of the surrounding fluid and any forces of attraction or repulsion which we may conceive of as exerted by matter without the volume on matter within it. Among these forces we have to reckon the weight

of the fluid contained in the volume, and, in most of the cases we shall treat of, this will be the only impressed force considered. Thus in this case the pressures on the boundary must balance the weight of the fluid in the volume. Suppose, then, that we consider the fluid in the volume to be solidified without any other change taking place, and find by the ordinary laws of statics the relations which must hold between the fluid pressure, which acts on the surface of this solid, and the forces applied to it from without in order that equilibrium may be maintained, it is clear that these same relations hold for the fluid.

In explaining the meaning of fluid pressure at a point, it will be noticed that nothing is said about the direction in which the area immersed in the fluid is to be placed, for it follows from the fundamental property of a fluid that the pressure is the same in all directions about a point, so that, if the small area be placed in a horizontal position, the force on it is exactly the same as if it were placed vertically with its centre at the same point of the fluid.

This may be proved as follows: consider a small tetrahedron[1] in the fluid having all its faces equal equilateral triangles, the forces excited by the surrounding fluid on its four faces will by symmetry be equally inclined to each other. These forces will balance the weight of the tetrahedron. Now, the weight of

[1] A tetrahedron is a solid figure with four plane faces, each of which is a triangle.

the tetrahedron is proportional to its volume, the force on any face is proportional to the area of that face, and, if the tetrahedron be made sufficiently small, the volume will be negligibly small when compared with the area of one of the faces. Thus the weight may be neglected compared with the forces on the four faces; hence these four forces are in equilibrium, and, since they are equally inclined to each other, it is necessary for equilibrium that they should be equal. Thus the forces on the four equal faces are equal when the tetrahedron is sufficiently small; and, since the faces are equal, the pressure at a point is the same for each face—that is to say, the pressure is the same for each of the four directions corresponding to the faces of the tetrahedron. The proposition may be proved in this way for all directions round any point.

Another fundamental property of a fluid is its capacity to transmit pressure equally in all directions. If by any means the pressure at any point of a fluid is increased, the pressure at all points is increased by the same amount. Thus, take a cylinder with a piston; place in it a portion of a solid which just fits the cylinder loosely, and put weights on the piston. The additional force thus applied to the top of the solid will be by it transmitted to the base; but unless the solid expands laterally under this force, so as to fit the cylinder more tightly, there will be no pressure on the sides. If the solid be plastic, and the weights added exceed a certain limit, it will give way and exert some

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pressure on the sides; if the substance in the cylinder be a fluid, the force exerted by the weights per unit of area on the surface in contact with the piston will be transmitted to every other unit of area in the fluid.

This is illustrated by Pascal's experiment. Imagine a closed vessel with a number of openings, into each of which a piston is fitted, and suppose all these pistons have the same area. Let the vessel be filled with fluid, and each piston be held in its place by a suitable force. On increasing by any amount the force on one of the pistons, it will be found that an exactly equal increment has to be applied to each of the others to maintain equilibrium. The increase of pressure on the one is transmitted equally to all the others. Or again, if the areas of the pistons be different, the additional forces required for equilibrium will be proportional to the areas, for the pressure—that is, the force per unit of area—is equally transmitted, so that the whole force exerted on an area is proportional to that area.

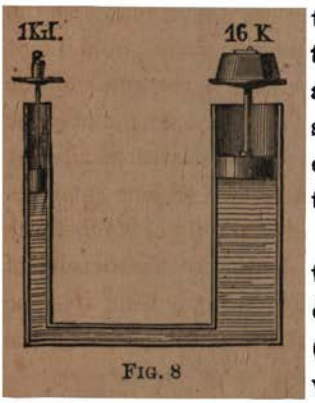

FIG. 8

The principle is turned to practical effect in the hydrostatic press, which consists (fig. 8) of two cylinders of very different diameters connected at their bases. Pistons fit into the cylinders, which are filled with fluid. On applying force to

the smaller piston the pressure at each point in the fluid is increased. This increase of pressure is transmitted equally to all points of the surface of the larger piston; the total force exerted on this will thus be equal to the increment of pressure multiplied by the area of the large piston, but the increase in pressure is equal to the force applied to the small piston divided by its area. Thus, by the action of the press the force applied is increased in the ratio of the area of the large piston to that of the small. In practice, of course, owing to the friction of the pistons in the cylinders, the force obtained falls short of its calculated value; but the nearer we can approach to the theoretical considerations implied in the above, the more closely the observation and calculation agree.

Density.—The density of a homogeneous body is given by the ratio of its mass to its volume. If the body is not homogeneous, its density varies from point to point. The density at any point is the ratio of the mass of a small volume, taken so as to include the point, to the volume when this is made so small that throughout it the body may be treated as homogeneous.

Specific Gravity.—The specific gravity of a substance is defined as the ratio of its density to the density of some standard substance such as water; thus, if ρ' be the density of water, ρ that of the substance, then the specific gravity $= \rho/\rho'$.

The term relative density is sometimes used instead of specific gravity. Other definitions of specific gravity

which lead to the same result as the above are the ratio of the weight of the substance to the weight of an equal volume of water, or the ratio of the mass of the substance to the mass of an equal volume of water; for, let W, M, W′, M′ be weights and masses respectively of a volume, V, of the substance and of water, then we have

$$\frac{W}{W'} = \frac{Mg}{M'g} = \frac{M}{M'} = \frac{\rho V}{\rho' V} = \frac{\rho}{\rho'} = \text{specific gravity}$$

in accordance with the first definition.

Hence, to find the specific gravity of a body—weigh it in air and then in water, the ratio of the weight in air to the loss of weight in water gives the specific gravity referred to water.

It should be noticed that in all the above cases specific gravity is defined as the ratio of two quantities of the same kind. It is therefore a number and does not depend on the units of measurement. The specific gravity of mercury is 13·596 whether we measure in pounds and feet or in grammes and centimetres.

To be quite accurate we should state the temperature at which the density of the water is to be taken, for this varies with the temperature and has a maximum value, as we shall see, at about 4° C. This, then, is the standard temperature to which reference is made. At this temperature the mass of a cubic centimetre of water is nearly 1 gramme—more strictly 1·000013 grammes—and thus the density of water at 4° C. is 1·000013 grammes per cubic centimetre, or practically unity; this being the case, the numbers expressing specific gravity

and density when measured in grammes per c.c. are the same. The specific gravity of mercury is 13·596, its density is 13·596 grammes per c.c.

With other units this relation no longer holds. The mass of a cubic foot of water is approximately 62·4 lbs.; thus the density of water is 62·4 lbs. per cubic foot, that of mercury is 13·596 × 62·4 or 848·4 lbs. per cubic foot.

There are many other ways of finding specific gravity. We will mention one which is applicable to a liquid. Weigh a body in air, in water, and in the liquid. The loss of weight in water is the weight of a volume of water equal to that of the body. The loss of weight in the liquid is the weight of the same volume of liquid; the ratio of these two, then, is the specific gravity of the liquid.

We may give as another illustration of the action of fluid pressure the following, which, with some modifications, forms a useful method of comparing the densities of two fluids. Bend a piece of glass tube into the shape of a U with long vertical legs, each about 50 centimetres long, and put water in it until the legs are each about half full. Then pour oil or some liquid which does not mix with the water into one of the legs. The common surface of the oil and water will fall in this leg and the water will rise in the other leg, but it will be found when all has come to rest that the oil and water do not stand at the same level in the two legs. Let us consider the reason of this and on what it is that the relative heights of the two depend.

Let A D B (fig. 9) be the U-tube, A the common surface of oil and water, A E the oil, A D B the water. Let A D be a horizontal line through the common surface of oil and water. We know that the pressure at a point in a given fluid is the same at all points in the same horizontal plane, and A and D are both in the water; thus the pressure at A is equal to that at D. Now, the pressure at A is equal to the air pressure at E together with the weight of a column of oil of unit section and of height A E; the pressure at D is the air pressure at B together with the weight of a column of water of unit section and of height B D.

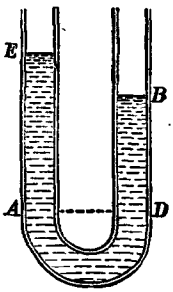

FIG. 9

Hence the weight of an oil column of height A E is equal to the weight of a water column of height B D, each column being of unit area in cross section.

But since the cross sections are the same the volumes of the two columns are proportional to their heights, and since the weights are the same the volumes are inversely proportional to the densities; hence the heights of the two columns are inversely proportional to their densities. Thus, if ρ, h be the density and height of the oil, ρ', h' the density and height of the water, we have $\rho h = \rho' h'$ or $\rho/\rho' = h'/h$: we can thus compare the densities by measuring the heights of E and B above the common surface. A U-tube, similar to the above, forms for many purposes a very convenient manometer

or pressure measurer; for, suppose that the lower part of the tube contains a fluid, water, say, or mercury, and that one end of the tube, A, say (fig. 10), is connected to a reservoir of gas or steam, the pressure of which it is required to measure. If this pressure is greater than that of the atmosphere, the liquid column will be depressed in this tube and raised at B in the other. Let a horizontal line through A cut the liquid in the second tube at C. Then the pressure at C is equal to that at A, both being points in the same fluid at the same level, and the pressure at C exceeds that at B by the weight of a column of liquid of unit area and of height B C. Thus, if h is the height B C, and w is the weight of unit of volume of the liquid, the pressure in the reservoir at A exceeds that of the atmosphere by $w\,h$ units of weight, thus the pressure is often conveniently measured by the height of the column of some given liquid which it will support.

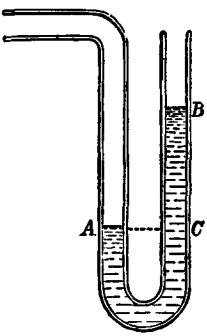

FIG. 10

If the pressure in the reservoir is less than that of the atmosphere the end A rises and B falls; this is the case with the barometer; for, suppose all the pressure is removed from the end A, then the height of the column A B measures the pressure on the end B, that is in this case the atmospheric pressure.

A barometer or instrument for measuring the pressure of the air may take various forms; the following is

the one in which standard barometers are usually constructed. A glass tube, about 34 inches long, closed at one end, is filled with clean dry mercury; the open end is then temporarily closed with the thumb or otherwise, and the tube is inverted in a cistern of mercury, care being taken to prevent the ingress of air. On again opening the lower end, and holding the tube vertical, the mercury falls somewhat in the tube and remains at a height of about 30 inches, or 760 millimetres above the mercury in the reservoir. The space above the mercury in the tube contains no air. The only substance there, if the instrument is well made, is mercury vapour, and this exerts on the top of the column an extremely small pressure. The pressure of the atmosphere acting on the free surface of the liquid in the reservoir is transmitted by it to the mercury in the tube and balances it. Thus the atmospheric pressure is measured by the weight of a column of mercury of unit cross section, and of height equal to that of the barometer. A simple experiment will prove that it is the atmospheric pressure which balances this column, for fit up a barometer with its cistern in the receiver of an air pump; on working the pump and exhausting the receiver the column falls, while on readmitting the air to the receiver it rises again.

FIG. 11

If we measure the height of the column we can calculate the pressure exerted by the air; for suppose the column be 76 centimetres high, then since 1 c.c. of mercury weighs 13·596 grammes, a column of mercury 1 square centimetre in area and 76 centimetres high, weighs 76 × 13·596, or about 1033 grammes. Thus, when the barometer is at this height the air presses on every square centimetre with which it is in contact with a force equal to the weight of 1033 grammes, that is rather over a kilogramme.

Or again, since the weight of one gramme contains 981 dynes, the pressure of the air per square centimetre is 981 × 1033 or 1,013,373 dynes. This is rather greater than a megadyne.[1] If we had been measuring in pounds and inches we might have shown that the pressure of the air was about 14·7 lbs. weight per square inch, that is a column of mercury 1 square inch in section and 30 inches high weighs 14·7 pounds. If the barometer had been made of some other liquid than mercury, its height would have been much greater, for mercury is much denser than any other liquid, and therefore, to balance a given pressure, a less column is required. Thus, since the specific gravity of mercury referred to water is 13·596, the height of a barometer column of water would be about 76 × 13·596, or 1,033 centimetres. The pressure which the air thus exerts is due to its weight, and so this 1033 grammes weight

[1] It will be remembered that the prefix *mega* stands for one million. A megadyne = one million dynes.

gives us the weight of a column of air one square centimetre in section extending from the surface of the earth to the upper surface of the air. If we could suppose that the density of the air did not change as we ascended, we might calculate the height of the atmosphere, for we know that the mass of 1 cubic centimetre of air at the surface of the earth with the barometer at 760 and the thermometer at freezing-point is ·001293 grammes, and since the mass of the column of air in question is 1033 grammes, its height, if of uniform density, would be 710,000 centimetres or 7·1 kilometres. This is sometimes called the height of the homogeneous atmosphere. The height to which the air extends is in reality far greater than this, for the density of the air is not uniform but decreases as we ascend. The density of the air is found by weighing a glass globe of known volume, which can be exhausted of air, first when full, and secondly when empty. From these observations the mass of air which fills the globe is found and then the ratio of this mass to the volume gives the density.

If a barometer be carried up a mountain, the mercury falls, showing that there is less pressure on the fluid in the reservoir. The mass of air above the barometer gets less as we ascend, and hence the weight which balances the mercury column gets less also and the column falls. We can obtain a formula giving a relation between this difference in the barometer reading and the difference in level of the two stations at

which observations are taken. Thus, observations of the height of the barometer at the top and bottom of a mountain may be used to find the height of the mountain. If we suppose that the difference in level is so small that we may neglect variations in the value of g, and call p and ρ the pressure and density of the air at the lower station, h the barometric height there, and h' that at the upper station; then, assuming the temperature to be the same at the two points of observation, we can show that the difference in level is equal to

$$\frac{p}{g\rho} \log \left(\frac{h}{h'}\right).$$

CHAPTER VIII

PROPERTIES OF FLUIDS—VISCOSITY, COHESION, CAPILLARITY, COMPRESSIBILITY

In the chapter on solid bodies we have considered at some length the elastic properties of such bodies, and when distinguishing between solids and fluids a preliminary explanation of the term viscosity was given. We must now investigate more closely this and some other properties of fluids. The fundamental property of a fluid is that, when at rest, it can only exert normal pressure on any surface with which it may be in contact. This proposition is also nearly true for many fluids, even though they be in motion, and most of the science of hydrodynamics is concerned with an ideal perfect fluid for which it holds always. But with any actual fluid the proposition is only approximately true; if we stir the water in a tumbler with a spoon, thus causing it to rotate, and then remove the spoon, the water continues to rotate for a little time, and then comes to rest. Now, if there were only normal pressure exerted by the glass on the water this would not happen; the water would rotate continuously instead of gradually stopping.

Again, suppose a sphere be suspended in a fluid by a vertical wire, and then caused to oscillate by twisting the wire, the sphere will come to rest in time owing to the frictional force which is brought into play between it and the fluid. It is found that a certain amount of force is required to shear a fluid, but the force does not depend on the amount of shear, as in a solid, but on the rate at which the shear is produced; if sufficiently long time be allowed for the process the very smallest force will suffice.

Thus, consider a fluid flowing along a canal, and take two plane horizontal sections, A B, C D (fig. 12), at distance a apart; let the velocity of the fluid along A B be v_1, and along C D be v_2; the fluid beyond these two sections exerts normal pressure on the fluid within the portion A B, C D. But besides this there is a tangential force due to the viscosity exerted parallel to the planes; the amount of this force is found to be proportional to $(v_1 - v_2)/a$, when the two sections are sufficiently close together and the difference between v_1 and v_2 not too large. If the tangential traction per unit of area of either section be $k(v_1 - v_2)/a$, then k is called the coefficient of viscosity.

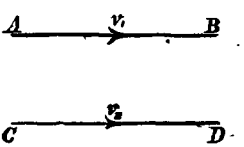

FIG. 12

If v_1, v_2 had stood for the displacements of the particles in the planes A B, C D in the direction of these lines, then $(v_1 - v_2)/a$ would measure the shearing strain, and since velocity is rate of displacement we see that $(v_1 - v_2)/a$

measures the rate at which shearing strain is being produced; thus the force arising from viscosity is proportional to the rate at which shearing strain is being produced; and the coefficient of viscosity is measured by the ratio of the tangential stress per unit of surface to the rate of shear. The viscosity of water is very small, being at 0° C. equal to ·018. Thus, if we take two planes at a distance of 1 centimetre apart, with water between them, and suppose that the water is at rest over the one plane, and moving with a velocity of 1 centimetre per second over the other, there will be a tangential pull of ·018 dynes on each square centimetre of either plane. It seems probable, as the result of experiment, that when water is flowing past a solid, the film of water in contact with the solid is at rest relatively to it; if the solid be at rest so is the water film; if the solid be moving the water film is moving at the same rate. If, then, we take the case of a solid at rest, with water streaming past it, it is clear that on moving outwards from the solid into the stream, there is a very rapid change in the velocity parallel to the solid; and as the force due to viscosity depends on this change, near the solid the force is considerable, but since the coefficient of viscosity is small the force is small in other parts of the fluid, where the change of velocity in directions at right angles to that of the flow is small. Thus, for a substance such as water, the effects of viscosity are small except near the surfaces of solids in the fluid. Hence, if we have water flowing through

a pipe or tube, it is only when the tube becomes small that the flow is much affected by the viscosity. The mathematical investigation shows that the quantity which flows through a tube of circular cross section of given length with a given difference of pressure between the ends is proportional to the fourth power of the radius; this result has been verified by the experiments of Poiseuille, and it is from these experiments that we infer that the film of fluid in contact with the walls of the tube is at rest. Some experiments of Helmholtz and Pietrowski lead to conclusions opposed to this, but a recent repetition of these experiments at the Cavendish Laboratory would seem to show that their results must have some other explanation. We can obtain a value for the coefficient of viscosity by measuring the flow through a tube of known radius and length when under known pressure; for, if p be the difference of pressure between the ends, l the length, and r the radius of the tube, it can be shown that the quantity which traverses any section of the tube per unit of time is given by $\pi p r^4/8 l k$, where k is the coefficient of viscosity. This quantity of fluid can be measured, and also the values of p, r, and l, and then the above formula gives the viscosity.

If a system be suspended by a vertical wire and then made to oscillate about the axis of the wire, the oscillations gradually decrease in amplitude. This decrease is due partly to internal friction of the wire, partly to the viscosity of the air. By properly

arranging the apparatus, measurements of the rate of decay of the oscillations can be made to give us a determination of the internal friction, or of the viscosity of the air or other fluid surrounding the system. Experiments of this nature on the viscosity of air were made by Maxwell. His oscillating system, shown in fig. 13, consisted of a number of parallel glass plates. These were suspended, with their planes horizontal, by a vertical steel wire. Between each pair of moving plates there was a fixed plate, the wire to which the moving plates were attached passing freely through holes in the fixed plates. The radii of the plates were considerable, and the distance between the plates small in comparison. If we consider the layer of air between two plates, an upper one moving, say, and a lower one

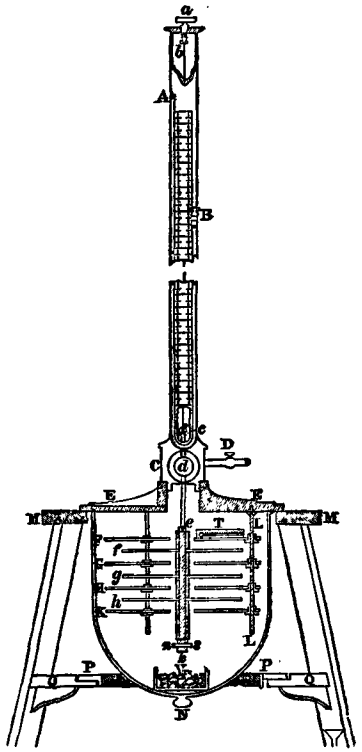

FIG. 13

fixed, the upper surface of the layer is attached to the oscillating plate and moves with it; the lower surface is fixed to the fixed plate. There is thus considerable difference in velocity between the upper and lower surfaces of the layer, which are themselves not far apart, and thus the effects of viscosity are large, and become measurable. The whole of this apparatus was contained in a large glass receiver, from which the air could be exhausted, and the connection between the viscosity and the density determined. Maxwell showed that the viscosity of a given gas is independent of the density, but increases with the temperature.[1] For air at the freezing-point Maxwell gives the value 0·001878 for the coefficient of viscosity. It is probable, however, that this value is too large.[2] Mr. H. Tomlinson gives as the result of his experiments that at a temperature of $t°$ Centigrade the coefficient of viscosity of air is

$$0{\cdot}0017155\ (1 + {\cdot}002751\ t),$$

and this is in close agreement with the values found by Mayer, Puluj, Obermayer, and others. In Mr. Tomlinson's experiments the oscillating system consisted of two cylinders, or two spheres, rigidly suspended from a light horizontal bar; this was supported by a

[1] More recent experiments, specially those of Holman with the temperature (*Phil. Mag.* vol. xxi.), have shown that the rate of increase is not quite so large as that given by Maxwell.

[2] The error in Maxwell's result seems probably to be due to an error of calculation, through which a wrong value has been used for the area of the plates.

wire of well annealed copper, great care being taken to secure freedom from draughts and uniformity of temperature. The rate of decay was observed as in Maxwell's experiments.

The result that the viscosity is independent of the density has an important theoretical bearing. It will be noticed that the viscosity of a gas increases as the temperature rises; that of a liquid, on the other hand, falls. Thus for water, according to Poiseuille, at a temperature of $t°$ C.,

$$k = \cdot 017793\ (1 - \cdot 0337\, t),$$

while Professor Osborne Reynolds has found that for olive oil

$$k = 3 \cdot 265\, e^{-\cdot 0128t}$$

Professor Reynolds has also pointed out that the assumed law of resistance for a viscous fluid, viz. that the resistance is proportional to the rate of shear, is only to be justified by actual experiment—by showing, that is, that we do get a constant ratio between the stress and the rate of shear in a viscous fluid for very different rates. This is found to hold so long as there are no sinuosities or eddies in the motion. Such sinuosities are formed even in channels with exactly parallel walls, according to the experiments of Professor Reynolds, when the velocity of flow exceeds a certain value, depending on the nature of the fluid and on the size of the channel. When they are formed the law of resistance

changes, becoming approximately proportional to the square of the velocity instead of the velocity.

When two portions of matter come into close and intimate contact, certain forces between them are brought into action which are insensible even when the particles are separated by very minute distances indeed.[1] Portions of the same substance are said to cohere, and the forces called into play are said to be forces of cohesion. If two different substances are brought close together so as to stick, they are said to adhere, and the forces between them are those of adhesion.

Substances, solid and fluid, hold together in consequence of these forces of cohesion between their molecules. Consider now a particle of a fluid well within the substance of the fluid; it is surrounded on all sides by other particles, each exercising its attractive force on it, and these various forces balance among themselves; also, since the forces of cohesion are only sensible at very small distances, this balance holds almost up to the boundary of the fluid. Let us suppose there is a limiting distance for each particle at which the molecular force in question ceases to be sensible, and let us draw round the particle, as centre, a sphere with this distance as radius. Then, so far as this molecular action is concerned, the particles outside the sphere do not

[1] Quincke has found that these forces cease to be sensible if the distance exceed about the twenty-thousandth part of a millimetre.

influence the central particle, and therefore this equilibrium balance of the forces holds so long as all parts of the sphere are within the body of the liquid—that is, since the radius of the sphere is very small, for all particles except those in a very thin skin close up to the surface. But now take a particle so close to the surface that its sphere cuts the surface. Then it is clear that there will be a resultant force on the particle, and we can show that this resultant force acts normally to the surface. The effect of this resultant force in cases in which it acts towards the fluid, as it will if the force between the molecules is attractive, is to increase the pressure at all points within the fluid, and the pressure at any point just within the fluid will depend in part on the form of the surface in the neighbourhood of that point. Thus, in consequence of this, the fluid particles possess more energy when near the surface of the fluid than when in the interior; but this increase of energy exists for only a very thin skin of the fluid. Now we have seen that in all cases the potential energy of a material system tends to become as small as it can consistently with the other conditions. But the particles near the surface will lose energy if they get into the interior, and the excess of energy due to the superficial particles will depend on the amount of surface. If, then, the amount of surface can be reduced, other things remaining equal, the energy of the system will be reduced, and, since in virtue of the general law the potential energy tends towards a minimum, the system

will endeavour to set itself so that the surface is as small as possible. This tendency may, of course, be opposed by gravity or other forces acting on the liquid, and the form actually assumed will be that in which the whole energy is a minimum. We can show readily that the surface of a sphere is less than that of any other solid figure having the same volume. Hence, if we could destroy the effect of gravity, a drop of water or mercury would take the form of a sphere. If we place a very small drop of mercury on a flat clean sheet of glass, it is nearly spherical in shape. Now it possesses potential energy, partly because some particles are higher than they need be,—we can conceive of the drop being all flattened down into close contact with the glass,—partly because of its surface. The flattening process would decrease the potential energy due to gravity, and therefore gravity tends to flatten the drop. But this same process would increase the surface, and thus increase the superficial potential energy. The form assumed is the one in which the whole energy is as small as possible.

The effects of this superficial energy are shown most strikingly perhaps when the surface of the fluid is very large compared with its volume, *e.g.* when the fluid is drawn out into a thin film. Thus, to maintain a soap bubble it is necessary that the pressure of the air within the bubble should be greater than that outside. If a bubble be blown in the ordinary way from a tobacco pipe, on withdrawing the pipe from the mouth the

bubble contracts, and the air it contains is expelled. In a thin film such as a soap bubble the number of particles which are in proximity to the surface bears a much larger proportion to the whole number in the film than is the case when the bubble contracts to a drop of soapy water, and thus the superficial energy is much greater in the bubble form than when the whole is a drop. Work has to be done to make the bubble, and this work is stored up in the superficial energy. The film behaves as if it were a stretched elastic membrane. If we think of any line traced on the bubble, it is as if the two portions of the film on either side of this line were pulling at each other with a certain tension. A simple experiment will illustrate this, and will give us the relation between this surface tension and the superficial energy.

Let A B C D be a piece of clean wire bent so as to form three sides of a rectangle, and let A D be a loose piece forming the fourth side. Hold A D in position, as in the figure, and make a soap film over the rectangle. On releasing A D it will be drawn up to B C. The film contracts, thus drawing the two together; now let T be the tension on each unit of length of A D, and let E be the energy of each unit of area of the film. Let A D = a and A B = b. The area of the film is $a\,b$ square centimetres, and its surface energy is E $a\,b$. The force on A D

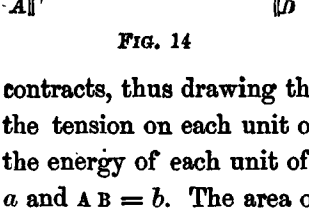

FIG. 14

is Ta, and the work done by this force in drawing AD to BC is therefore T$a \times b$. But this work has been done at the expense of the surface energy, and therefore Tab = Eab or T = E. That is, the tension per unit of length is equal to the surface energy per unit of area.

The value of T has been measured by various methods, and we shall have more to say on this point. One way in which it might theoretically be done, though in practice it would be difficult to carry it out, would be by determining what was the greatest weight that could be supported from AD. Let this weight be W, then Ta = W or T = W/a. It was found by Plateau that the surface energy of a soap film was about 55 ergs per square centimetre. According to this, since the film has two surfaces, the tension of each surface of the soapy water is 27·5 dynes per centimetre.

Quincke has shown that for pure water in contact with air the value of the surface tension is about 75 dynes per linear centimetre.

The above all turns on the supposition that the fluid particles have more energy when near the surface than when in the interior. This need not be the case. It is quite possible that the reverse should hold, and then the fluid sets itself to expose as great a surface as possible. Thus, the surface energy of oil in contact with either air or water is very small, the sum of the two, viz. for oil and air, oil and water respectively, being less than that for air and water. If, then, a drop of oil be placed on water, we have three surfaces to consider, viz.

air and water, air and oil, oil and water. As the oil spreads over the water, the area of the first surface is lessened, that of the other two is increased, but by the spreading the whole surface energy is decreased, for the decrease of the air and water energy is greater than the increase of the other two; hence the oil does continue to spread out over the water, becoming thinner and thinner, till it at last contains so few molecules in its thickness that it ceases to have the properties of a liquid in mass. A similar result happens when a drop of water is placed on a clean glass plate. We have in this case three surfaces, air and glass, air and water, and water and glass. The superficial energy of the air and glass surface is greater than that of the other two; hence the total energy is decreased if the water spreads over the plate, and this accordingly happens. But in some cases a drop of one fluid may be seen floating on the surface of another. Let us inquire what it is which determines the form of the drop, and the angles between the three surfaces.

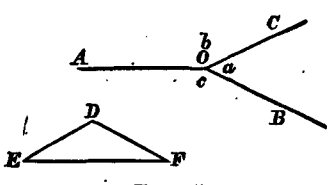

FIG. 15

Take a point O (fig. 15) on the curve in which the substances a, b, c meet, and let OA, OB, OC be the three surfaces. Then along each of these surfaces there is a tension; thus the point O is acted on by three forces, T_a, T_b, T_c, along these three lines, and these forces are in equilibrium; but it is known in this case, that

if a triangle be drawn, having its sides parallel to the forces, the forces are represented by the sides. If, then, we know the values of the three tensions, and take a line EF parallel to OA, and equal to T_a, and on this line as base describe a triangle DEF, such that its sides FD and DE are equal to T_b, T_c; then the form of the drop will be such that the lines OB, OC are parallel to FD and DE respectively. This condition, then, determines the angle between the two surfaces of the drop. If the values of T_a, T_b, and T_c be such that the triangle cannot be constructed, the drop cannot be formed. In the case of oil on water, if we take *a* as the oil, *b* as the air, and *c* as the water, then since $T_b + T_c$ is less than T_a, we cannot describe the triangle, and the drop cannot be formed. The same is the case with water on the clean glass. In both these cases the angle between the two surfaces of the film vanishes; the water is said to wet the glass; if a little mercury, on the other hand, be placed on a clean glass, it will contract to a drop, and the mercury surface will make a definite angle of contact with the glass; various observers have measured the value of this angle. Again, suppose we have a film of water on a level piece of glass, and allow a drop of alcohol to fall on the centre of the water film, it will be found that the water at once flows away from the centre in all directions; where it has been brought into contact with the alcohol the tension is much reduced, and the greater tension of the exterior portions produces this apparent rupture. The same phenomenon accounts for the motions

of small bits of camphor floating on water; the camphor dissolves in the water, thus reducing the surface tension, and then the difference in tension sets up motion in the water film which draws the camphor with it.

Again, if a clean piece of glass be dipped in water the surface of the water does not remain horizontal but rises, as at the point A in figure 16, where it comes in contact with the glass. The water wets the glass and tends to cover it completely, for by this the superficial energy is decreased, but in so doing it is raised itself above its former level, and this process increases its potential energy due to gravitation; so long as the decrease is greater than the increase the water will rise; the height it will reach will be determined by making the whole energy a minimum.

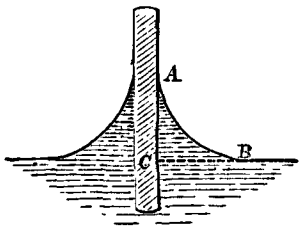

FIG. 16

Suppose that A B is the curved surface of the water, B being the point where it becomes horizontal. Let B C be a horizontal line and consider the forces acting on the raised portion of the water; the total force will of course depend on the length of glass which is immersed. Let us deal with a part of unit length at A. The water surface is vertical, and thus the surface tension, T, acts vertically, pulling the water up. At B, on the other hand, the tension is horizontal. In addition to these tensions the water is acted on by its weight and by the

horizontal pressures on the surface A C. Let m grammes be the mass of water raised per unit of length of the immersed surface. Then the weight raised is mg dynes, and thus $T = mg$ dynes per centimetre; the value of m depends on the form of A B, and this can be calculated from a knowledge of the air pressure though it is beyond our limits to attempt it.

Suppose now we have a second plate and we immerse it parallel to the first. The same weight of water will be raised by it so that the total weight raised will be 2 T dynes per centimetre of length immersed and will remain the same whatever be the distance between the plates. When the plates are some distance apart a portion, B B′ (fig. 17), of the water surface between them will be horizontal and at the same level as originally, but if the plates be moved up together, so that, if the surfaces A B, and A′ B′ remained unaltered, B would come to the right of B′; then clearly in order that the sum of the volumes A B C, A′ B′ C′ may remain unaltered A and A′ will have to be raised. When the plates are brought still closer together the volume of the curved part at the top will be small compared with the whole volume raised, and if d be the distance between the plates, and h the height of A or A′ above the undisturbed surface, then the volume raised per unit of length of immersed surface is hd;

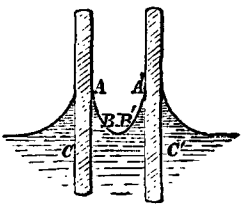

FIG. 17

and if ρ be the density of the liquid, the mass raised is $h\,d\,\rho$, thus $T = h\,d\,\rho\,g$.

The surface tension T can be calculated by measuring h and d. If the two vertical plates be not parallel but inclined to each other at a small angle, then the height to which the water rises will be different at different points and the line of contact of the water and glass will be curved. Let O A C B (fig. 18) be one of the plates, and suppose that it is in contact with the other along the vertical line O A. Let P be any point on the water surface, and let P N be vertical and meet in N the undisturbed surface, and P M horizontal meeting O A in M; then there is between the plates a sort of wedge of water, and the distance between the plates increases uniformly as we go along O B, being, at any point such as N, proportional to O N. Now the height to which the water is raised is inversely proportional to this distance. Hence P N is inversely proportional to O N, or the area P N O M is the same for all positions of P on the surface. A curve which has this property is called an hyperbola; thus the form of the curve of contact in this case is an hyperbola.

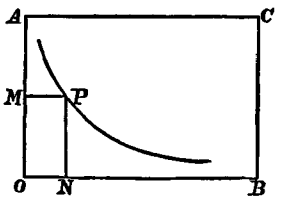

FIG. 18

The bodies which are immersed in the liquid need not, of course, be plane, and the effects are shown markedly by glass tubes of narrow bore. Let r be the

radius of such a tube placed vertically in a liquid which wets it, and let h be the height which the water rises in the tube. Then this column of water is supported by the tension acting along the curve in which its upper surface meets the tube, and this tension acts vertically; the length of the curve, since it is a circle of radius r, is $2\pi r$, so that the whole vertical force is $T \times 2\pi r$. The water raised is a cylinder of height h and radius r; thus its volume, neglecting the curvature of the upper surface, is $\pi r^2 h$. Hence the weight of liquid raised is $\pi r^2 h \rho g$ dynes. Thus $2\pi r T = \pi r^2 h \rho g$.

$\therefore T = \frac{1}{2} r h \rho g$ dynes per centimetre; or, putting the result in another form, $h = \dfrac{2T}{r \rho g}$

In the first form the result can be used to calculate T. We see also that the height to which a given fluid can rise is inversely proportional to the radius of the tube. The smaller the tube, the greater the rise: thus these effects are most marked in tubes of small bore. Such are known as capillary tubes, from 'capillus,' a hair—and the various phenomena we have been discussing are classed together as phenomena of capillarity.

If the liquid does not wet the tube or the plate, its surface, where it is in contact with the plate, will not be vertical, and the surface tension will act in a direction inclined to the vertical at a definite angle. We shall have then to deal with the vertical component of the tension instead of the tension itself. If the angle between the surface of the liquid and the upper portion of the

immersed solid be less than a right angle the liquid will be depressed by the contact and the surface tension will hold it down. The surface of the liquid will be convex to the air and the pressure inside will be rather greater than that outside; in the cases hitherto dealt with the reverse is the case—the liquid is concave to the air and the tension reduces the pressure. Mercury in contact with glass is an example of the latter case. The form of the surface when a glass plate, A B, is immersed in mercury is as shown in fig. 19, while if a narrow tube, C D, be dipped into mercury the mercury is depressed as in the figure. In the case of fig. 19 an upward thrust is exerted on the glass plate by the capillary action. This upward thrust may sometimes be sufficient to retain, supported on the water, a body which would sink if it were once totally immersed beneath the surface. Thus a needle, which of course sinks in water, may be made to float. The water does not wet the steel and so a depression is produced in the surface. Various forms of insects can move over the surface of water, being supported by the same action.

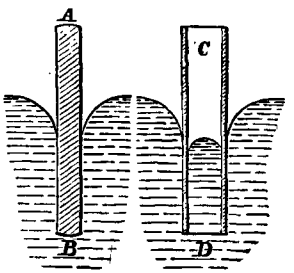

FIG. 19

Again, if small bits of cork or other light material, which is wetted by water, be floating in water in a vessel, it will be noticed that when they approach within a

small distance of each other, or of the sides of the vessel, provided it be not filled to the brim, they rush together or to the side of the vessel in the two cases respectively; this is also due to capillarity; for, consider two parallel plates in a fluid which wets them—the water is, as we have seen, raised between the plates, and the pressure at any point in this raised part of the fluid is less than the atmospheric pressure; for the pressure at all points at the same level is the same; thus, if Q (fig. 20) be a point between the plates at the same level as the undisturbed surface of the fluid, the pressure at Q is the atmospheric pressure, and if P be a point in the raised portion, at a higher level, that is, than Q, the pressure at P is less than that at Q. Thus, a point on one side of the plate is acted on by a pressure less than that due to the atmosphere, while a corresponding point at the same level on the other side is exposed to the atmospheric pressure; there will thus be a force on each plate tending to draw it closer to the other; this force will, of course, only come into play when the plates are so close together that the liquid between them is raised, and will increase as the plates approach. The small pieces of cork or other substance wetted by the fluid are acted on just as the plates here described; if on the other hand the plates were not wetted by the fluid, but were placed close together so as to cause a

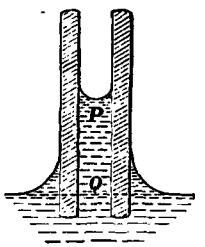

FIG. 20

depression instead of an elevation, there would then be a force tending to separate them.

The difference between the pressure in the fluid and the pressure just outside the surface depends on the curvature of the surface film, being greatest when the curvature is greatest. When two plates are very close together, the free surface of water between them is very much curved, and the pressure inside is very small compared with that outside. Hence, if a drop of water be placed between two plates which are close together, there is only a small pressure acting on the glass in contact with the water, while on the opposite of the glass there is the full atmospheric pressure; hence the two plates of glass adhere with considerable force.

One or two methods for determining the value of the surface tension have been mentioned already; we will, in conclusion, just refer to two others. The principles on which the form of a drop of mercury resting on a glass surface can be calculated have been explained. Measurements of the shape actually assumed by such a drop can be applied to calculate the surface tension. This has been done for water, mercury, and other liquids by Quincke. When a piece of clean glass or of thin platinum foil is suspended in water or some other liquid which wets it, the water is raised by the surface tension, and of course the glass or foil is pulled down with an equal force. By suspending the foil from one arm of a delicate balance, and determining the apparent increase of weight thus caused, the total down-

ward pull is obtained, and from this the surface tension can be calculated.

While all fluids differ from solids in offering no permanent resistance to stresses tending to change their form, a liquid has considerable elasticity of volume, and the application of great stress is necessary to produce even a very small change in volume. The quantity k defined on p. 80, that is, the ratio of the pressure required to produce a given cubical contraction to the contraction produced, is very large; in a gas this quantity depends upon the pressure and is comparable with it. Experiments have been made on the compressibility of liquids, but they are difficult to conduct, for, in order to allow for the changes of volume of the containing vessel produced by the pressure it is necessary to know the elastic constants of the actual material of which it is composed; and these cannot usually be obtained with great exactness. Some form of piezometer is employed for the purpose; in Oersted's apparatus there is a glass bulb with a narrow vertical tube attached. The bulb and part of the tube are filled with the liquid to be experimented on (say water), the bulb is placed uppermost with the open end of the tube dipping into a vessel of mercury. This mercury fills the part of the tube which is not occupied by the water. The whole is placed in a strong closed glass vessel so as to be visible and the vessel is filled with water; a force pump is attached to this receiver, also a gauge for measuring the pressure;

the mercury serves merely to keep the water in the bulb separate from that in the receiver and to transmit the pressure produced by the pump to the contents of the former; the volumes of the bulb and of a given length of the tube are known. On increasing the pressure by the pump the mercury is seen to rise in the tube; if it were true that the volume of the interior of the bulb were not altered by the pressure applied to the two surfaces of the glass, then the decrease in volume of the water produced by the pressure could be calculated from this rise of the mercury, but as a fact the effect of the pressure on the bulb is to cause it also to contract, and the observed decrease in volume of the water is the difference between the actual decrease and the decrease in the volume of the bulb. Now this decrease is just the same as that of a solid sphere of glass of the same size as the interior of the bulb and to which the same pressure is applied, for we may consider the sphere as a sort of vault made up of a large number of similar cubical bricks, and we have seen that where hydrostatic pressure is applied to such a cube, it contracts to a smaller size but remains similar in shape to itself; thus the sphere when compressed is made up of the same number of similar bricks as before, but each brick has been reduced in size and the whole vault is thus reduced in size by a proportionate amount. Thus, to determine the compressibility of water from these experiments, we require to know that of the material of the vessel. Various endeavours have been made to

avoid this difficulty, but their success has not been great, for they depend on conditions being satisfied as to the shape and thickness of the bulbs, and as to the uniform nature of the material in order that the theory necessary to deduce the required results may be applied, while in practice these conditions are almost unrealizable.

Since the volume of a fluid varies with the temperature, in experiments on the compressibility of fluids care has to be taken to keep the temperature constant.

The compressibility of water decreases at first as the temperature rises, according to the recent experiments of Pagliani and Vincentini, and reaches its minimum value at about 63° C.

The following table is quoted by Tait ('Properties of Matter,' p. 189) from Cailletet's results. These have been corrected for the compression of glass by the use of Regnault's value, viz. ·00000184 per atmosphere.

Substance	Pressure in atmospheres	Average compression per atmosphere
Water at 8° C.	705	·0000469
Sulphuric ether at 10° C. . .	630	·0001458
Bisulphide of carbon at 8° C. .	607	·0000998
Sulphurous acid at 14° C. . .	606	·0003032

Taking, then, the average compressibility of water to be at 10° approximately $\cdot 5 \times 10^{-4}$ per atmosphere, and remembering that a pressure of an atmosphere is roughly a megadyne or 10^6 dynes per square centimetre, we find that the compressibility is $\cdot 5 \times 10^{-10}$ and the value

of k, the modulus of compression, is the reciprocal of this, or 2×10^{10} dynes per square centimetre. Under this pressure the density of water would be doubled; the pressure is equivalent to about 20,000 atmospheres. Tait[1] has also found that the relative compressibility of sea water to pure water is ·925.

The value of the modulus for mercury at 15° is about 56×10^{10} dynes per square centimetre, or about twenty-seven times as great as that of water.

[1] Many other details as to the results of other experimenters' work are given by Tait, *Properties of Matter*, chap. x.

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CHAPTER IX

GASES—THE GASEOUS LAWS

A GAS, we have seen, differs from a liquid in that it is very easily compressed, and that, if the pressure be sufficiently reduced, it is capable of indefinite expansion. A rise of temperature causes it to expand in volume, or if this expansion be prevented the pressure is increased. At ordinary pressures the density of gases is very small compared with that of water. Thus the mass of a litre of air at atmospheric pressure and a temperature of 0° C., is about 1·29 grammes, that of a litre of water is 1,000 grammes. Hence water is about 800 times as dense as air at this pressure and temperature.

The general relation between the pressure and volume of a gas at constant temperature was first stated in his 'Defence of the Doctrine touching the Spring and Weight of the Air,' published in 1662, by the Hon. Robert Boyle, one of the original members of the Royal Society of London. He showed that within the limits of accuracy allowed by his apparatus, the pressure of a given mass of gas at constant temperature is inversely proportional to its volume; thus, by doubling

the pressure, the volume is halved; and conversely, if the volume be doubled the pressure is halved; or, to put it in symbols, if p be the pressure of a given mass of gas at constant temperature and v its volume, then pv is a constant.

Since the volume of a given mass of gas is inversely proportional to its density, it follows that the pressure is proportional to the density, and Boyle's law[1] is sometimes stated in the form that the pressure of a gas at constant temperature is proportional to its density. Boyle proved the law to be true for pressures both greater and less than one atmosphere. For the first case the apparatus used by him was a bent glass tube (fig. 21), with one limb much longer than the other. The shorter limb is closed at the top, the longer one being left open. A little mercury is poured into the bottom of the tube so as to enclose the air in the short limb, care being taken that the level of the mercury should be the same in the two limbs, when the tube is placed in a vertical position. When these conditions are satisfied we have a mass of air in the short limb under atmospheric pressure. Let us now suppose that the shorter tube is uniform in cross section and that the volume corresponding to each centimetre of its length

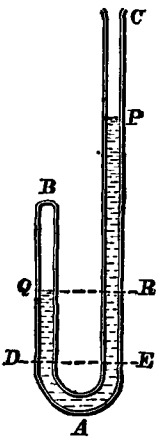

FIG. 21

[1] The law is known as Mariotte's law in France.

is known. Scales are attached, and by reading the height of the mercury in the short limb, the volume of air enclosed can be calculated.

We have thus a known volume of air at atmospheric pressure. More mercury is then poured into the open limb, and it is observed that the level in both branches of the tube rises, but at unequal rates, the level in the open limb being above that in the other. Thus, suppose that after a time the levels have reached P and Q, A B being the closed limb and A C the open one. Let a horizontal line through Q meet the other column in R, and suppose that initially the columns in the two tubes were at D and E respectively; let h be the height of the mercury barometer.

The pressure at Q is equal to that at R, both being points at the same level in the same fluid. The pressure at R exceeds that at P by the weight of a column of mercury of height P R. The pressure at P is given by h, the height of the barometer. Thus the pressure of the enclosed air is measured by a mercury column of height $h + \text{P R}$. Originally the pressure was measured by a height h only.

The volume of the air originally was proportional to B D; when the mercury stands at P Q it is proportional to B Q. Now it is found experimentally that these volumes are inversely proportional to the pressures, that is, that

$$\frac{\text{B D}}{\text{B Q}} = \frac{h + \text{P R}}{h},$$

or

$$h \times \text{B D} = (h + \text{P R})\,\text{B Q}.$$

It is necessary of course for this that the temperature should not alter during the experiment. Thus Boyle's law was proved to be true at any rate for the pressures greater than one atmosphere employed by him. For pressures less than an atmosphere a different form of apparatus is better. A vertical tube with a cistern at the top is employed; this is filled with mercury. A long glass tube is inserted in this, being allowed to rise somewhat above the level of the mercury. In this position the upper end of the tube is hermetically sealed, and we have thus a volume of air enclosed at atmospheric pressure. The glass tube is now raised, and it will be found that the mercury also rises with the tube, though not so fast as the tube itself. Thus the enclosed air expands. Let EF (fig. 22) represent the tube in its new position; let P be the level of the mercury in the tube, R the level in the cistern, and Q the point at which the mercury stood when the tube was depressed at first. The volume of the mass of air originally was proportional to EQ; in the second condition it is proportional to EP; the pressures in the two cases respectively are measured by h and $h - PR$; on making the various observations it is found that the equation $h \times EQ = (h - PR) \times EP$ is satisfied, or that Boyle's law holds for pressures less than one atmosphere.

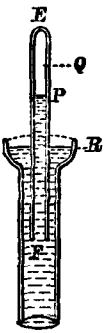

FIG. 22

The results of such experiments are very conveniently represented in the graphic manner already

made use of. Let us take two lines at right angles and measure off volumes along the horizontal line O x, pressures along the vertical line O y (fig. 23).

Thus, if O N represent the volume and O M the pressure, and through N and M two lines N P and M P be drawn, vertical and horizontal respectively, to meet at P, then the point P represents the condition of the gas as to pressure and volume, and as these change we shall get a series of positions for P forming a continuous curve. If the condition under which the pressure and volume vary be that the temperature remains constant, such a curve is called an isothermal line, and for a gas obeying Boyle's law, the curve has the property that the area O N P M is constant for all positions of P. Such a curve is called a rectangular hyperbola, and has the form P P′ shown in fig. 23.

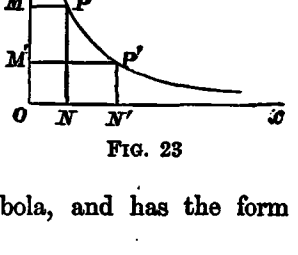

FIG. 23

The diagram is called a diagram of energy because by means of it we can measure the work done by the gas in expanding. For let P′ be any other point on the curve, and let P′ N′ be drawn parallel to O y to meet O x in N′, then it can be proved that the work done by the gas in expanding from a volume O N to a volume O N′ is measured by the area P P′ N′ N.

Suppose the gas contained in a cylinder with a piston. Let the piston be A square centimetres in area

and let the pressure of the gas be p. Suppose now that the piston is withdrawn a very small distance h, so small that for this change we may neglect the change in pressure it causes. Then the force on the piston is p A, and the distance which the point of application of this force moves is h; thus the work done by the force is p A h; now A h clearly measures the increase in volume occupied by the air, so that if v_1 be the initial volume, v_2 the final, we have A $h = v_2 - v_1$ and the work done is given by $p(v_2 - v_1)$; thus, if in a diagram of energy N′ be near to N while O N represent v_1 and O N′ v_2, the work done is the area of the parallelogram P N N′ P′. Thus, if the volume change by a finite amount, the work done is the sum of a series of elongated parallelograms such as the above, and the sum of such parallelograms becomes ultimately the curved figure, P P′ N′ N.

The same result may be shown to hold even though the gas is not in a cylinder with a frictionless piston. This follows at once from the fact that no force is needed to alter the shape of a mass of gas without altering its volume, and therefore if the gas be contained in some irregular-shaped vessel no work is required to transfer it to a cylinder of the same volume; it can then be allowed to expand to the new volume, and the work done will be measured as above, while to transfer it from the cylinder as enlarged to the irregular vessel it is to occupy finally requires no energy, since the volumes of the cylinder in the second condition and of the final vessel are the same. It should be noticed that the work

done is represented by the above area even though the expansion be not isothermal. Only in this case the curve P P′ is no longer a rectangular hyperbola. Again, it follows from the above that if the pressure and volume of a gas undergo any series of changes, finally coming back to the initial condition, that series of changes is represented on a diagram of energy by a closed curve such as P Q R (fig. 24), and the work done during the changes is given by the area P Q R; if the curve has been described in the direction P Q R, work has been done by the gas in the cycle; if on the other hand the curve is

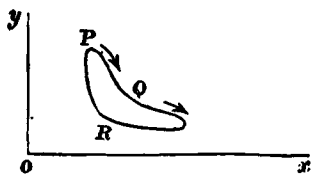

FIG. 24

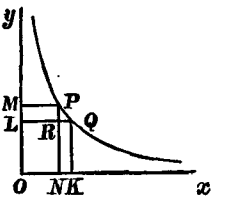

FIG. 25

described in the direction P R Q, work is done on the gas by the external pressures applied. In the first case heat or energy in some other form has been supplied to the gas to enable it to expand and do the work; in the second, the gas will have given up energy in some form to the surrounding bodies. Such a series of changes as the one just described in which the working substance is in the end restored to the same condition as it was initially is called a cycle.

In the case in which the relation between the pressure and volume is that given by Boyle's law, it is easy

to show that the resistance to compression of the gas is equal to the pressure.

For let P, Q (fig. 25) be two points on the curve, and let P M, Q L be parallel to the line O x of no pressure, P N, Q K to the line O y. Also let P N meet Q L in R. Then, since $p\,v$ is constant, the area P M O N is equal to Q L O K. Take away the common part O L R N, then the parallelogram L M P R is equal to the parallelogram N R Q K; that is,

$$\mathrm{MP} \times \mathrm{PR} = \mathrm{QK} \times \mathrm{QR},$$

or

$$\frac{\mathrm{PR}}{\mathrm{QR}} \times \mathrm{MP} = \mathrm{QK}.$$

Now k, the resistance to compression or extension, is measured by the ratio of the change in pressure to the compression or extension produced; the change in pressure is P R, the actual change in volume is R Q, and the original volume is P M. Thus the cubical compression is measured by Q R/P M and k, the resistance to compression, is the ratio of P R to this. Hence, for a gas obeying Boyle's law,

$$k = \frac{\mathrm{PR}}{\mathrm{QR}} \times \mathrm{PM} = \mathrm{QK} = p.$$

According to this it appears that at a given pressure all gases are equally compressible. Despretz in 1829 was the first to show that this was not true, and therefore that Boyle's law could not hold for all. This result is most readily shown by means of Pouillet's apparatus (fig. 26), which consists of two long glass tubes of very uniform bore and with thick walls. The tubes, T, T, are

fastened tightly to an iron reservoir containing mercury, and this communicates with another vessel of mercury, into which a screw plunger fits. Two different gases are introduced above the mercury in the two tubes which are then sealed off. Let us suppose the apparatus so adjusted that the mercury stands at the same level in the two tubes, so that the gases are under the same pressure. On lowering the plunger the pressures on the gases are increased, and it will be observed that the two columns of mercury do not rise by exactly equal amounts in the tubes. The two gases are therefore differently compressed, and clearly if the one obeys Boyle's law the other cannot.

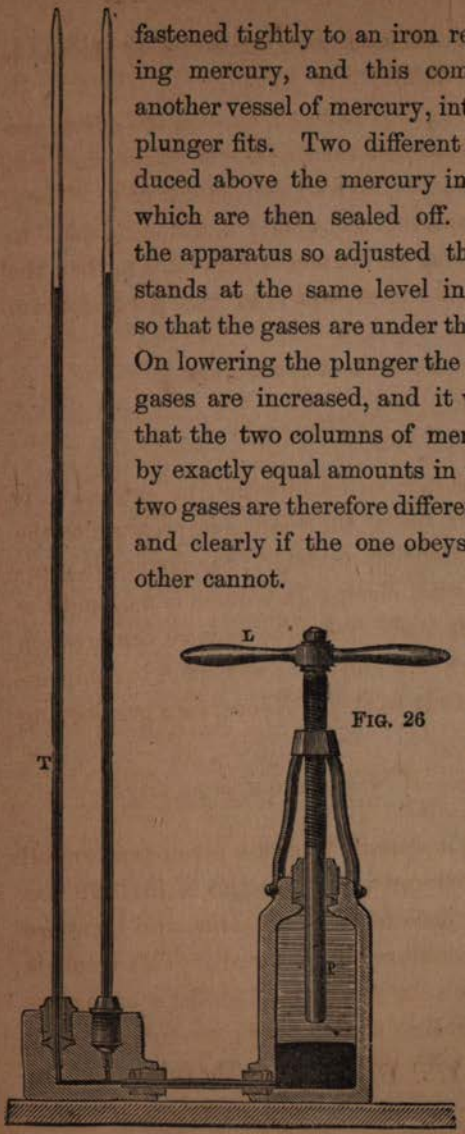

FIG. 26

The difference in the heights of the two columns is extremely small for gases such as oxygen, hydrogen, nitrogen, and others, which are not readily liquefied, while for gases such as carbonic acid or

ammonia, which at ordinary temperatures can be reduced to the liquid state by pressure only, it is marked.

It became important, then, to investigate whether any of the permanent gases obeyed Boyle's law exactly, and if not, what the variations from it were. This problem was undertaken by Regnault, who used a modification of Boyle's apparatus, the long tube A C of fig. 21 being about 2,500 centimetres in height and being fitted in a small tower in the Collège de France at Paris. Instead of pouring mercury into the open end of the tube at C, a force-pump was attached at A, and by means of it the mercury was forced into the tubes. Regnault took a quantity of gas at a measured pressure and then determined the increase of pressure required to compress the gas to half of its original volume. He found that for all the gases except hydrogen, up to the limits to which he could work, the pressure required to reduce the volume by a given amount was rather less than that given by Boyle's law; or, stating the same fact in a different way, the reduction of volume produced by a given pressure is greater than that given by the law; the various gases are more compressible than they would be if the product of the volume and pressure were constant; hydrogen, however, is an exception, being less compressible. It is important to notice that in such experiments the temperature of the gas must be kept constant; this was done in Regnault's experiments by surrounding the shorter tube by a

large tube containing water. With the length of tube employed by Regnault the maximum pressure obtainable was about 25 atmospheres, and under these conditions the product $p\,v$ becomes less as the pressure is increased for all gases except hydrogen; for hydrogen it increases steadily with the pressure. It was shown, however, by Natterer that for very high pressures such as that due to 2,000 atmospheres the product $p\,v$ is for air and nitrogen greater than it should be according to the law. Thus at these high pressures air and nitrogen behave like hydrogen, being less compressible than they would be if the law held. These results of Natterer were confirmed by Cailletet who showed that for nitrogen the product $p\,v$ decreased until the pressure due to about 60 metres of mercury was reached, while for greater pressures the product $p\,v$ again increases. The following table gives some of Cailletet's results, the temperature being about 15° C. The pressure is given in metres of mercury.

p	$p\,v$	p	$p\,v$
30·359	8,184	79·234	8,162
49·271	8,022	144·241	8,966
59·462	7,900	181·985	9,330

But the most complete contribution to our knowledge is to be found in the results published by Amagat. The long column of mercury by which his pressure was produced and measured was contained in

a steel tube which was placed in a pit shaft 400 metres deep, the short tube containing the air being at the bottom of the shaft. The mercury was pumped into this tube as in Regnault's experiments by a force pump. The gas used in Amagat's experiments with the mercury column was nitrogen. Having once determined the compression for this gas corresponding to definite pressures, the values for other gases, oxygen, hydrogen, &c., could be obtained by a comparison between them and nitrogen, as in Pouillet's experiments. Amagat found that for nitrogen, air, oxygen, carbonic oxide, and ethylene the product pv decreases at first as the pressure increases, and then increases, the pressure at which it is a minimum being different for the different gases. The following table gives some of his results for nitrogen:

p in metres of mercury	pv	p	pv
20·740	50,989	82·970	51,226
35·337	50,897	128·296	52,860
47·176	50,811	190·855	55,850
55·481	50,857	252·353	59,921
61·241	50,895	327·388	65,428

The results for the other gases except hydrogen are similar to the above, but in the case of carbonic oxide and ethylene the variations in pv are much greater than for nitrogen. For carbonic oxide the value of pv varies from 26,325 at a pressure of 24 metres to 22,915

at 133 metres, and up to 29,289 at 304 metres; while for ethylene the minimum value is 9,370 at 64 metres and the maximum obtained 29,333 at 303 metres. In the case of hydrogen pv increases uniformly with the pressure. Amagat has represented his results graphically by plotting a curve in which the vertical ordinates

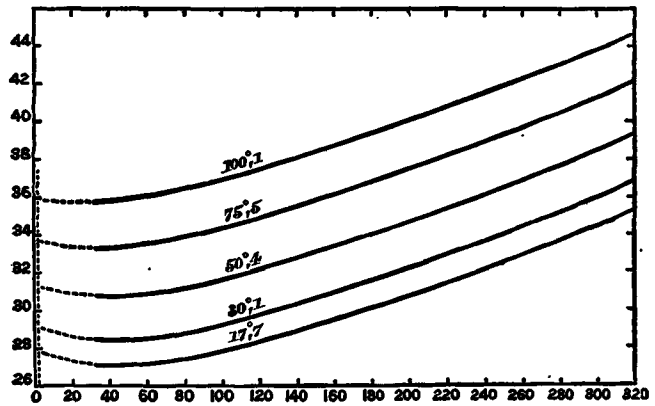

FIG. 27

represent the values of pv while the abscissæ give the values of p.

If Boyle's law were true such a curve would be a horizontal straight line. In reality this is not the case, but the curves for nitrogen have the forms given in fig. 27, taken from Amagat's paper in which he has further investigated the question for a series of different temperatures for each gas. For hydrogen the curves are a series of straight lines showing at all temperatures a

gradual increase of pv with p, while for carbonic acid the variations in the value of pv, and also the pressures corresponding to its minimum value are very marked. In all such cases in which pv is not constant the isothermal curve will not be a rectangular hyperbola.

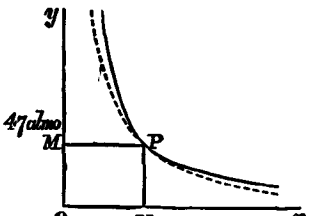

FIG. 28

Thus, for nitrogen, for which the minimum value of pv is at a pressure of about 47 metres, the isothermal curve will take the form of the strong line in fig. 28, while the hyperbola is indicated by the dotted line.

When we come to pressures much less than one atmosphere the experiments are even more difficult, because of the extreme shortness of the column of mercury which has to be measured; and the results of various experimenters are not in accord with each other.

So far we have been dealing with the isothermal lines of gases, and have supposed that the substances in question retain the gaseous form through all the changes to which they are subject, but there is a class of bodies which in some of their properties resemble gases, and which yet differ from a gas widely in other respects.

Thus, on applying heat to water, we raise its temperature to its boiling point, and on further application

of heat the water takes the form of steam. Now suppose we have a mass of water and steam at 100° C. in a closed space, and can maintain it at that temperature, then the pressure of the steam will be that due to 76 centimetres of mercury, and that whatever be the volume of the space occupied by it; if we increase the volume, more steam will be formed, and in order to maintain a constant temperature heat must be supplied, but the pressure will remain unchanged. If we reduce the volume some steam will be condensed, and to maintain a constant temperature heat must be removed from the mass, but the pressure will be the same. Or again, suppose we have a space filled with steam at 100°, but at a less pressure than that due to 76 centimetres of mercury; there will of course be no water present in these circumstances. Now let the volume be decreased, the pressure will increase, approximately in accordance with Boyle's law, though somewhat more rapidly than if the law held strictly, until a pressure of 76 centimetres is reached; after this there will be no further increase in pressure, the isothermal line therefore will be horizontal, until the whole is liquid. When this state is reached a very small change of volume will be accompanied by an enormous rise of pressure, and the line will be nearly vertical. Similar changes take place at other temperatures, though for them the pressure exerted by the steam when in contact with its liquid and constant is not 76 centimetres, but less or greater according to the temperature. Thus, the isothermal lines of steam, at

any rate for temperatures between zero and a maximum of several hundred degrees Centigrade, divide themselves into three parts. The first of these, starting from a considerable volume, as shown in fig. 29 by the curve P′ Q′, is nearly a rectangular hyperbola; falling below it as the pressure and volume corresponding to condensation are reached at Q; then comes a horizontal straight line Q R; for this part the steam is being condensed to water, and then a nearly vertical line R S for the liquid itself. Steam is said to be a vapour as distinguished from a gas, though as yet we have not made the exact line of demarcation between vapours and gases clear.

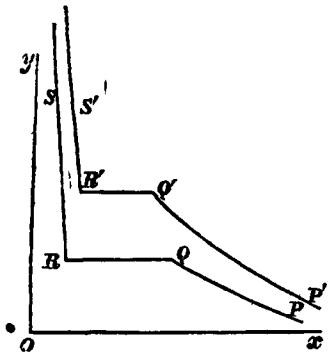

FIG. 29

If a quantity of liquid such as water, ether, alcohol, and many others be placed in a closed space, it is found that a portion of this liquid takes the gaseous form; it is said to evaporate, and heat is required for the process. The vapour formed increases the pressure already existing within this space, the pressure due to the vapour being added to the original pressure, but this increase only goes on up to a certain limiting value depending on the temperature. When that limiting pressure has been reached no more vapour is formed, the evaporation

ceases, and the space is said to be saturated with the vapour; the maximum or saturating pressure of the vapour depends on the temperature. The following table gives its value for water vapour in millimetres of mercury for a few temperatures:

Temp.	Pressure	Temp.	Pressure
0	4·60	40	54·91
10	9·17	60	148·79
15	12·7	80	354·62
20	17·39	100	760·00

The numbers given were obtained by Regnault.

Thus, if we had a large space filled with aqueous vapour at a low pressure and at a temperature of, say, 40°, and reduced the volume of the space, the pressure would increase up to the value 54·91, and after that there would be no further increase of pressure but some of the vapour would become liquid.

A vapour can be reduced to the liquid state by pressure alone without a change of temperature.

The change from vapour to liquid can be produced by reducing the temperature instead of by altering the volume. Thus, suppose the temperature of the mass of vapour is reduced, its pressure, like that of a gas, is somewhat lowered by this process.[1] But after a time a temperature is reached at which the space is saturated with the vapour; a further reduction of temperature causes condensation to the liquid form. The tempera-

[1] The laws of this change we shall investigate shortly.

ture at which a space is saturated with aqueous vapour is called the dew point.

Now nearly all gases have been liquefied by various experimenters, and many have been reduced to the solid form, but to succeed in this it has been found requisite in many cases both to subject the gas to an intense pressure and also to reduce the temperature enormously. It is found that for all gaseous bodies there is a certain temperature, called the critical temperature, which has the following property: If the body be above the critical temperature, it cannot be liquefied by pressure only; if it be below the critical temperature, pressure alone is sufficient to change it to the liquid form.

Thus Dr. Andrews, to whom we owe the first accurate investigation of this part of our subject, gives the following critical temperatures:

Carbonic acid	30·92
Ether	187·5
Alcohol	258·7
Carbon bisulphide	262·5
Water	411·7

Hence at a temperature above 30°·92 carbonic acid is a gas and cannot be liquefied by pressure; at temperatures below this an increase of pressure is sufficient to change the state from gaseous to liquid. A gas is a vapour when it is below its critical temperature. Thus at ordinary temperatures steam is a vapour, its critical temperature being 411°.

On the other hand, the critical temperatures of the so-called permanent gases are extremely low. They are very difficult to determine, but it seems probable that the value for oxygen is about —130° C. and for nitrogen about —167° C. As we have already stated,

FIG. 30

our knowledge of the properties of the critical temperature is largely due to Andrews' researches on carbonic acid; before giving an account of them, however, we will refer to some earlier experiments in which the methods used to produce great pressure were very simple.

Faraday's apparatus (fig. 30) consisted of a very strong bent glass tube. In one end of this tube was

placed a substance which on the application of heat emitted the gas to be experimented on. The other end of the tube dipped into a freezing mixture of ice and salt; the gas set free was liquefied in the cold end of the tube by its own pressure. In later experiments he forced the gases by means of a pump into glass tubes contained in a mixture of solid carbonic acid and ether; by this means all the gases known to him except oxygen, hydrogen, nitrogen, marsh gas, and carbonic oxide were liquefied. Shortly after Thilorier invented an apparatus for producing readily large quantities of liquid carbonic acid.

FIG. 31

The researches of Andrews, to which we shall again refer shortly, on this substance were conducted with the apparatus shown in fig. 31, which in its action resembles that of Pouillet. The gases which Faraday failed to liquefy yielded at the lower temperatures employed by Cailletet and Pictet. They both made use of the fact that the rapid expansion of a gas requires the absorption of heat and lowers its temperature. Cailletet at first employed a modification of Andrews' apparatus, having a stop-cock attached to the reservoir of water by which the pressure was produced. On turning this stop-cock the pressure was suddenly reduced; the gas, which had been compressed to about 300 atmospheres at a tem-

perature of −29° C., expanded and produced so rapid a fall in temperature that a jet of liquid was seen in the tube.

In Pictet's apparatus the gas was compressed in a narrow tube and then suddenly released as above. This narrow tube was surrounded by a second containing liquid carbonic acid, and was cooled by the rapid evaporation of the acid. This tube again was kept surrounded by liquid sulphurous acid, which by its evaporation reduced the temperature of the carbonic acid. In this apparatus hydrogen became solidified into a grey metallic substance.

Since the critical temperature of carbonic acid is, as we have said, 30°·92, it is a convenient substance to work with. Fig. 32 gives a copy of the diagram by which Andrews expressed his results, taken from Maxwell's 'Heat.' The dotted line was introduced by Maxwell, and shows the space within which the substance can exist either as a vapour or a liquid. The lowest isothermal in the diagram is that for 13°·1. For this temperature, and at pressures less than 48 atmospheres, the substance is a gas and the isothermal is approximately a rectangular hyperbola. On reaching a pressure of about 48 atmospheres—measured by the compression of the air in a second tube connected with the first—the gas begins to pass into the liquid form. As the volume is still further diminished, the pressure remains constant. The isothermal is a horizontal straight line until the whole is liquefied, when it changes again, becoming a

nearly vertical line. This line, it will be seen, agrees generally in form with that for steam at 100° C. In Andrews' apparatus the glass tube containing the gas can be surrounded by a wider tube in which water can be placed and the temperature of the gas can be varied by varying that of the water. On raising the temperature to 21°·5 we get the second isothermal shown, differing from the first only in the two facts that the pressure at which condensation takes place is now nearly 60 instead of 48 atmospheres, and secondly that the volume occupied by the substance when it begins to condense is much less than previously, while when the whole is liquefied the volume of liquid formed bears a much larger proportion to that of the gas at the same pressure and temperature than was the case for the lower temperature.

The next temperature shown is that for 31°·1, and in this case the substance never exists in two distinct forms in two parts of the tube. Between 65 and 73 atmospheres the isothermal is a good deal flatter than it would be for a perfect gas; from 73 to 75 atmospheres the substance changes in volume for a small change of pressure very rapidly, but by no means suddenly, while above this pressure the volume decreases less rapidly than would be the case for a gas, but much more rapidly than ordinarily happens with a liquid.

If, following Amagat, we plot, as in fig. 33, the value of pv in terms of p for this temperature, we shall find that it has a minimum somewhere about a pressure of

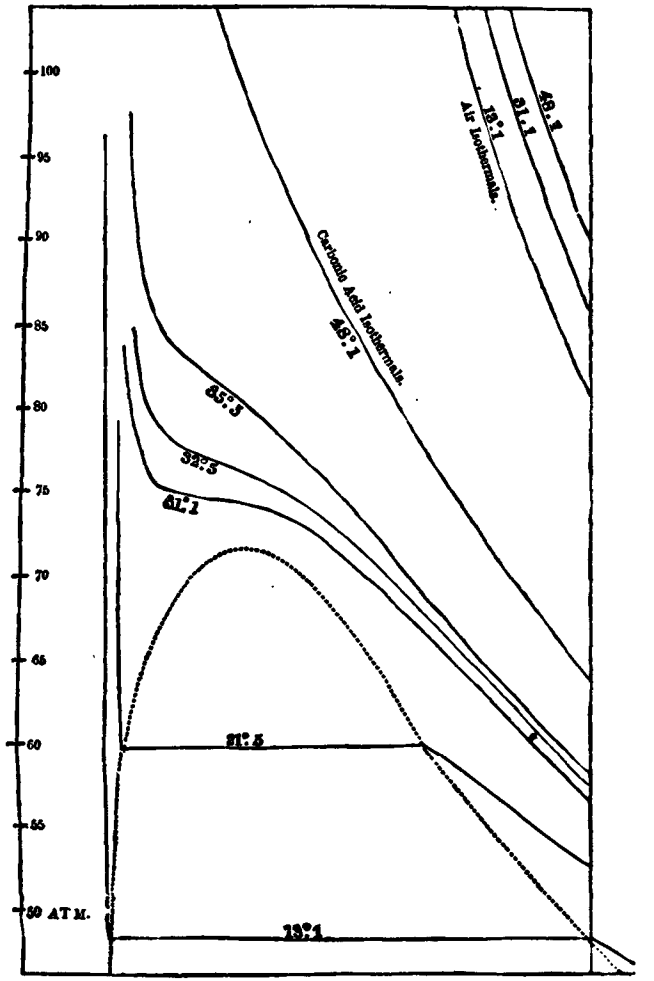
100
95
90
85
80
75
70
65
60
55
50 ATM.
Carbonic Acid Isothermals.
49°.1
Air Isothermals.
13°.1
21°.1
49°.1
85°.3
32°.3
21°.1
31°.5
13°.1

FIG. 32

74 atmospheres. As the pressure rises the value of pv decreases, and the decrease is much more rapid as we approach the pressure of about 74 atmospheres; from that point on pv increases somewhat rapidly with the pressure.

When the temperature is higher, the carbonic acid iso-

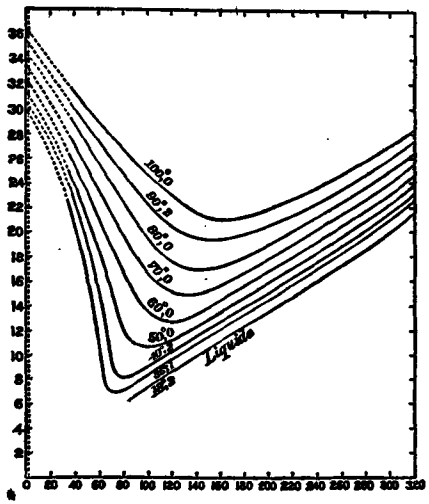

FIG. 33

thermals differ less and less from hyperbolas, and when the values of pv are plotted, as in Amagat's diagram, the dip indicating the minimum value of pv is less and less marked, though even at a temperature of 100° it is clearly visible. We may compare this with the corresponding diagram for nitrogen shown on p. 149, in which a

minimum value for pv is also shown, though it occurs at a lower pressure.

Returning to the Andrews diagram, let us consider how the isothermal corresponding to the critical temperature 30°·92, would run; it is not shown in the original figure, it just touches the dotted line, but does not cut it. The values of the pressure and volume corresponding to this point are called the critical pressure and volume; the point is called the critical point. When we have a substance at the critical point, *i.e.* at the critical temperature, and also of the definite pressure and volume indicated by this point on the diagram, it is extremely sensitive to very small changes in its condition. If the temperature fall ever so little, it can exist in the two distinct states of gas and liquid, each occupying a different part of the tube, and thus, while no separation of gas and liquid can be detected, 'small variations of pressure or temperature produce such great variations of density that flickering movements are observed in the tube, resembling in an exaggerated form the appearances exhibited during the mixture of liquids of different densities, or when columns of heated air ascend through colder strata.' Moreover, it is possible to transform the substance by proper treatment from a state in which it has undoubtedly the properties of gas to one in which it is equally undoubtedly a liquid without any sudden change of state. For, take some carbonic acid and raise its temperature till it is well

over 30°·92, say at 50° C., then compress it still at 50° C. till the pressure is about 100 atmospheres; all this time it has the properties of a gas. Now cool it, still keeping the pressure 100 atmospheres, till the temperature falls below the critical temperature, say to 20° C. At this temperature carbonic acid at a pressure of 100 atmospheres is a liquid; but during the last process no point can be noted at which a sudden change from gaseous to liquid has taken place; the transition has been continuous all the time.

So far we have been dealing with the laws of the expansion of a gas at constant temperature. It is true that we have considered the expansion of gases at various different temperatures; but in any one expansion the temperature has been constant, and we have not dealt with the change in volume or pressure produced by the application of heat and consequent change of temperature.

To discuss this completely it would be necessary to enter farther into the science of heat than our limits will allow; it is necessary, however, to refer to some few principal facts.

The fundamental law which connects together the volume and temperature of a gas is known as Gay Lussac's, or Charles's, law. It states that 'a gas at constant pressure expands by the same fraction of its volume at 0° C. for each rise of temperature of 1° C. measured by the mercury thermometer.'

This fraction is found to be approximately the same for all the permanent gases, and is very nearly $\frac{1}{273}$; let us denote it by a. Then if v_0 be the volume of the gas at the freezing point, $a v_0$ is the increase in volume for each degree of temperature, and hence for $t°$ C. the increment is $a v_0 t$. Thus, if v is the volume at $t°$ C.,

$$v = v_0 + a v_0 t = v_0 (1 + a t).$$

We shall refer again to the experiments by which this result was obtained; let us for the present consider a few of its consequences. If the temperature be lower than the freezing point, *i.e.* if t in the above equation be negative, the volume decreases at the same rate. Taking a as $\frac{1}{273}$, let us determine the volume at a temperature of -273. We have,

$$v = v_0 \left(1 + \frac{t}{273}\right) = v_0 (1 - 1) = 0.$$

Thus, at this temperature, and at atmospheric pressure, the gas would, if the laws still held, have no appreciable volume. This temperature is known as the absolute zero of the air thermometer and temperatures reckoned from it are called absolute temperatures. To find the absolute zero of a gas thermometer for which a is not exactly equal to $\frac{1}{273}$, we have to find a value for t, for which $v_0 (1 + a t)$ is zero; this gives us $1 + a t = 0$

$t = -1/a$. So that the absolute zero of the gas thermometer is $1/a$ degrees Centigrade below freezing,

and thus the temperature of the freezing point reckoned from absolute zero is $1/a$. Let us denote this by T_0, and let T be the absolute temperature corresponding to a temperature t on the Centigrade scale. Then T is found by adding to t the number of degrees between absolute zero and freezing point thus:

$$T = t + T_0 = t + \frac{1}{a}.$$

Now we have

$$v = v_0(1 + at) = v_0 a \left(\frac{1}{a} + t\right) = \frac{v_0 T}{T_0}$$

$$\therefore \frac{v}{T} = \frac{v_0}{T_0}.$$

Thus Gay Lussac's law may be stated in the form that the volume of a gas at constant pressure is proportional to the temperature reckoned from the absolute zero of the gas thermometer. We may make this rather clearer, perhaps, by a hypothetical experiment in the following manner. Suppose the gas to be contained in a long straight tube (fig. 34), and separated from the external air by a small pellet of mercury, and when the temperature is at freezing point, let the gas occupy 27·3 centimetres of the tube. Divide the space above this into millimetres or tenths of centimetres; then, by Gay Lussac's law, for each rise of temperature of 1°, the level of the column of gas will rise by one of these divisions, for one such division is $\frac{1}{273}$ of the volume of the gas at 0°. Each division also corresponds to a

degree Centigrade, and there are 273 divisions from freezing point down to the bottom of the tube. The volume of the gas is clearly proportional to the number of divisions between its upper surface and the bottom of the tube, and this same number of divisions measures the temperature in Centigrade degrees, reckoned, however, from 273 degrees below freezing point. Clearly also, since the gas contracts by one of these divisions for 1° Centigrade, it will contract to no volume if cooled 273 degrees below the freezing point. The temperature corresponding to the bottom of the tube will be the absolute zero, and the number of divisions between the top of the air column and the bottom of the tube will give the temperature on the absolute scale of the air thermometer.

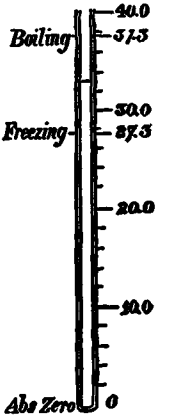

FIG. 34

This, it must be noticed, all supposes that the pressure remains constant. Let us now investigate the relation between the pressure, temperature, and volume, when all may vary. Let P_0, V_0, T_0, be the initial pressure, temperature and volume, and let us change T_0 to T, keeping P_0 constant. Suppose V_0 becomes V', then we have

$$\frac{V'}{T} = \frac{V_0}{T_0} \quad \therefore \quad \frac{P_0 V'}{T} = \frac{P_0 V_0}{T_0}.$$

Now keep T constant and change V' to V, P_0 in consequence becoming P. Then, if Boyle's law is assumed

as true, we have $P_0V' = PV$, and the above equation becomes

$$\frac{PV}{T} = \frac{P_0V_0}{T_0}.$$

This is the general relation between the three quantities; if we like to express it in terms of the temperature Centigrade, and a the coefficient of expansion, since $T_0 = 1/a$, and $T = 1/a + t$ we have $T/T_0 = 1 + at$ and

$$PV = P_0V_0\frac{T}{T_0} = P_0V_0(1 + at).$$

There are three cases to notice.

(1) If the temperature is constant, $t = 0$ and $PV = P_0V_0$.—Boyle's law.

(2) If the pressure is constant, $P = P_0$ and $V = V_0(1 + at)$.—Charles's law.

(3) If the volume is constant, $V = V_0$ and $P = P_0(1 + at)$.

Thus we see that, according to this, the pressure of a gas at constant volume increases uniformly with the temperature by the same fraction of the pressure at 0° Centigrade. Moreover this fraction is, if the assumptions made above are accurate, the same as the fraction of the volume which occurs in Charles's law; but we have already learnt that one of our assumptions, viz. Boyle's law, is not strictly true, and thus it is found as the result of experiment that, while the pressure can be expressed as in law (3), the coefficient a which occurs is not strictly the same as that in (2). We should more

correctly write $P = P_0 (1 + a' t)$ where a' is a very little different from a for gases which nearly obey Boyle's law.

Again, careful experiment has shown that a is not strictly the same for all gases, so that the zero of all gas thermometers is not strictly the same, although for the more permanent gases the differences are but small. Thus Regnault gives the following table.

Gas	a, coefficient for constant pressure	a', coefficient for constant volume
Hydrogen	·003661	·003667
Air	·003670	·003665
Nitrogen		·003668
Carbonic oxide	·003669	·003667
Carbonic acid	·003710	·003688
Nitrous oxide	·003719	·003676

Regnault has also shown that the value of the coefficient depends slightly upon the initial pressure of the gas at 0°. The above values are calculated for an initial pressure due to 760 mm. On increasing this to 3,655 mm. Regnault found as the value of a' for air ·003709.

It will be noticed that while for hydrogen a is less than a', the reverse is the case for the other gases; this is a consequence of the fact that within the range of pressure employed the deviations from Boyle's law are opposite in direction in the two cases: for hydrogen pv increases with p; for the others it decreases.

The terms, absolute temperature, absolute zero, &c., apply really to a method of measuring temperature

devised by Sir William Thomson. According to it the absolute values of two temperatures are to one another in the proportion of the heat taken in to the heat rejected in a perfect thermodynamic engine, working with source and refrigerator at the higher and lower of the temperatures respectively. It would carry us too far from our subject to discuss all that is implied in the above definition, or to attempt to give an account of the long series of experiments by which Thomson and Joule compared the scale of temperature thus defined with that of the air thermometer. Since, according to this scale, the measure of a temperature is independent of the nature of the substance which is used to measure it, the name absolute scale has been given to it. It was shown that the scale of the air thermometer is very nearly the same as the absolute scale, and the temperature of a mass of air reckoned from absolute zero is almost exactly in accordance with the above definition. Moreover these same experiments showed that while air, oxygen, nitrogen, and carbonic acid differed slightly from this absolute scale in one direction, so that the absolute zero of a constant pressure air thermometer is slightly above the true absolute zero, hydrogen differs in the opposite direction, giving an absolute zero which is really a very little too low; and on determining from the experiments the corrections the following results were obtained for the temperature of melting ice, reckoned from the true absolute zero, as determined by thermometers of hydrogen, air, and carbonic acid.

Gas	Uncorrected estimate of absolute temperature of melting ice	Correction given by observations to direct reading of thermometer	Corrected value of temperature of melting ice above true absolute zero
Hydrogen .	273·13	− ·13	273·00
Air . . .	272·44	+ ·70	273·14
Carbonic acid .	269·5	+ 4·40	273·90

Thus these numbers are very close together, and it is clear that Thomson's absolute zero of temperature, the temperature of a body which contains no heat, is very close to that of the air or hydrogen thermometers. A mass of air at this temperature would contain no heat. If it were compressed under constant pressure, the volume occupied would be inappreciably small; or if, the volume being kept constant, the pressure was being diminished, it would at this temperature vanish.

Up to the present we have dealt with the relation between the pressure, volume, and temperature of a gas, either generally or under one of three conditions, viz., temperature constant, pressure constant, volume constant. Corresponding to these we get three sets of lines on our diagram of energy—the isothermal lines or lines of equal temperature; for a perfect gas, a set of rectangular hyperbolas; the lines of equal pressure, a series of horizontal straight lines; and the lines of equal volume, a set of vertical straight lines. There is a fourth condition under which expansion or contraction may occur frequently, and which from its importance deserves some notice, though it must be very brief.

The condition in question is that no heat should be allowed either to enter or leave the substance. We may suppose the gas enclosed in a cylinder with a piston, the walls of the cylinder and the piston being made of non-conducting material. The condition is, we can show, satisfied in the very rapid contractions and expansions which occur in the propagation of a wave of sound through the air. It would take us beyond our limits to investigate the form of the curve which in this case would represent the relation between the pressure and volume on the diagram. We can, however, easily show that it somewhat resembles the isothermal line in its properties, but is at each point rather steeper than the isothermal through the same point. For, suppose a gas is compressed by a given amount, in general heat is produced in the process and the temperature is raised; if the compression is to be isothermal this heat must be removed; if the heat be not removed the temperature must rise; and since a definite compression is produced the pressure must be increased by this rise of temperature by a greater amount than that caused by the isothermal compression. Thus the increase of pressure arising from this contraction is greater than it would be if the contraction were isothermal; the elasticity is also greater and the curve is steeper. Expansion or contraction which occurs under such conditions is said to be adiabatic, and the curves on the diagram are called adiabatic lines. The further development of this part of our subject belongs

to the Science of Thermodynamics, and a full account of the important results which can be drawn from the study of these various lines will be found in Maxwell's 'Heat.'

We must now refer to the details of the experiments by which the fundamental laws were established.

Gay Lussac used a glass bulb of known volume (fig. 35), with a long tube attached. The tube was

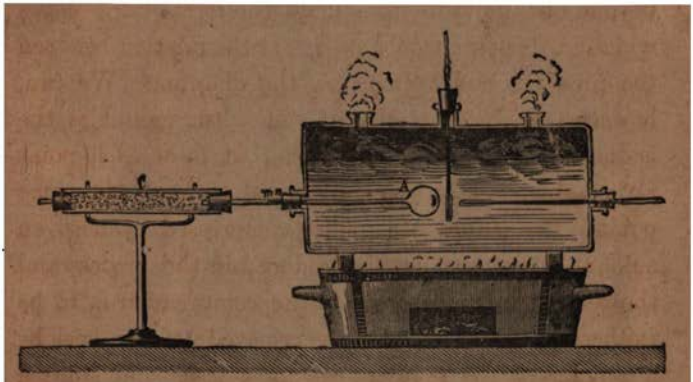

FIG. 35

graduated, and the volume corresponding to each division determined; a small pellet of mercury enclosed the air on which it was required to work. The bulb and tube were immersed in melting ice and the position of the mercury noted; they were then immersed in hot water, the temperature of the water being found by a mercury thermometer, and the position of the pellet again found. The first observation gave the volume of the air at 0°, the second that at the higher temperature.

Assuming the reading of the barometer to have remained unchanged from these results, the coefficient of expansion of air at constant pressure can be found. By the use of this apparatus Gay Lussac was led to enunciate his law; but the value, ·00375, which he found for the coefficient was too high.

For the determination of the increase of pressure at constant volume due to rise of temperature, Balfour Stewart's form of thermometer (fig. 36) is very useful. The air or gas is contained in a glass bulb; from this a glass tube of narrow bore projects upwards and the tube is bent twice at right angles and is fastened into a reservoir containing mercury; a second vertical tube is also fastened into the same reservoir, and there is a screw plunger by which the level of the mercury in the tubes can be adjusted. There is also a mark on the vertical part of the first tube just below the bend. The plunger is moved until the level of the mercury just reaches to this mark. When the bulb containing the air is immersed in melting ice, the pressure is found by observing by means of scales attached to the instrument the difference of level of the mercury in the two tubes, and adding this to or subtracting it from the reading of the barometer as may be required. We have thus a mass of air at known pressure and temperature. The bulb is now placed in the steam from boiling water and the air expands, driving the mercury up the second tube; but by the use of the plunger the mercury in the first tube can be brought back to its

original position, the pressure of course being thereby increased, and the mercury rising still more in the second tube. When all is steady the difference of levels is again determined, and from this the increased pressure can be found; by dividing the increase in

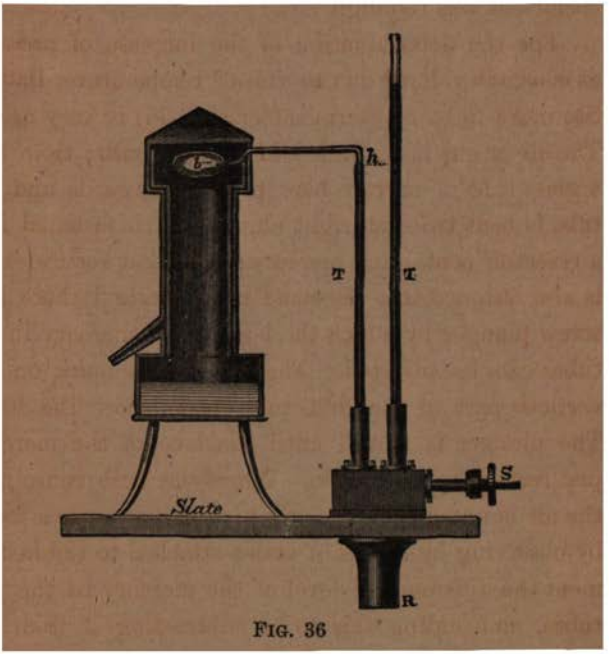

FIG. 36

pressure by 100, and by the pressure at 0°, we get the coefficient required. In accurate experiments corrections are required for the expansion of the bulb, and for the fact that a portion of the air in the narrow tube is not at the same temperature as the rest.

A similar apparatus can be used to measure the coefficient of expansion at constant pressure. In this case, however, it is necessary that the volume of the vertical part of the first tube should be about $\frac{1}{3}$ of that of the bulb. The preliminary adjustments are made much as before; but the air is allowed to expand into the vertical part of one tube, and the plunger is moved until the difference of level of the mercury in the two tubes is the same as it was in the first part of the experiment. The air has then expanded at constant pressure; but a large portion of it, that in the vertical tube, is at a different temperature to the air in the bulb, and so the whole of this requires to be surrounded by a jacket which can contain steam and hot water at the same temperature as the bulb. It was by means of an apparatus of this kind that Regnault's results were obtained.

CHAPTER X

THERMAL PROPERTIES OF BODIES—SPECIFIC HEAT—LATENT HEAT—MOLECULES

MANY other properties of bodies are so intimately connected both with our present subject and with the science of heat, that it is difficult to draw a distinct line between the two. We will mention one or two of special importance.

Experiment shows that the quantities of heat required to raise equal masses of different bodies through the same range of temperature are very different. Thus, if we take as the unit of heat the quantity of heat required to raise a gramme of water one degree, it is found that this amount of heat will raise about thirty grammes of mercury one degree, or that the quantity of heat required to raise a gramme of mercury one degree is only one-thirtieth of that required to raise the same mass of water.

The ratio of the quantity of heat required to raise a given mass of any substance one degree, to that required to raise the same mass of water one degree, is called the specific heat of that substance. The follow-

ing table gives the values of the specific heats of various substances:

Lead	·0314	Zinc	·0956
Platinum	·0324	Iron	·1138
Mercury	·0333	Glass	·18768
Silver	·0570	Ether	·503
Copper	·0951	Alcohol	·615

To be quite accurate in the above definition, the temperature at which the water is to be should be mentioned, and, instead of stating that the rise of temperature of the water is to be one degree, we should say definitely from 0° to 1°, for the specific heat of water at 50° referred to this standard is 1·0042.

In stating the values of the specific heat of a gas it is necessary to mention the physical conditions under which the gas is placed. Thus, if heat be applied, the temperature is raised, while at the same time the pressure and volume may be changed, and the heat required to produce a definite rise of temperature is found to be very different in the different cases which may arise. If the pressure be kept constant and the gas allowed to expand, the following values have been found for the specific heats:

Air	·2375	Carbonic oxide	·2450
Oxygen	·2175	Carbonic acid	·2163
Hydrogen	3·4090	Steam	·4805

If the volume be kept constant, the specific heats of the gases are all less than the above.

Let us consider briefly the reason for this difference.

In the first case, when the gas expands, it does work in overcoming the pressure of the atmosphere, the work done being measured by $p\,v$, where p is the pressure and v the increase in volume. To enable it to do this work it requires a supply of energy, and therefore more heat is necessary than is required to produce the same rise of temperature at constant volume. Again, since the volume has increased, the density of the gas has been reduced, and it is thus in a different state of aggregation. Now it is quite possible that, apart from the energy required to overcome the atmospheric pressure, energy may be necessary to thus change the state of aggregation of the gaseous molecules. It is conceivable that they are held together by inter-molecular forces, and that to move them further apart from each other work has to be done against these forces. We thus see two reasons why the specific heat at constant pressure may differ from that at constant volume: (*a*) because of the energy required to overcome the pressure of the external air; (*b*) because of the energy required to do work against the internal molecular forces. Now Joule showed that the energy necessary for the second purpose was in the permanent gases extremely small.

If a mass of air or hydrogen be allowed to expand against no external pressure, so that the work done under (*a*) vanishes, there is on the whole no change of temperature. Thus no heat need be supplied to keep the temperature constant, and the energy required under (*b*) also vanishes.

Joule's first experiment on this point was the following. Two copper vessels (fig. 37) were taken, each having a capacity of about 134 cubic inches; these were connected by a pipe with a tap between the two, and immersed in a reservoir of water, which was carefully stirred. Air at a pressure of about 22 atmospheres was compressed into one, while the other was exhausted, and the temperature of the water was taken by a delicate thermometer. On opening the stop-cock the air rushed from the one cylinder to the other, the water being stirred. No change in the temperature was observed, so that Joule concluded that no change of temperature occurs when air is allowed to expand in such a way as not to develop mechanical power.

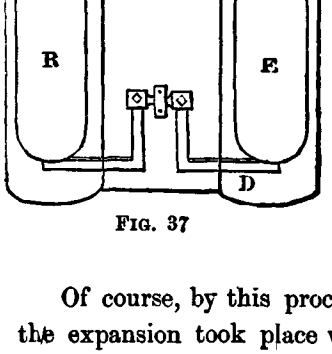

FIG. 37

Of course, by this process the cylinder from which the expansion took place was cooled, the air particles rushing from it gained kinetic energy at the expense of their heat, but the cylinder into which they rushed received thereby an equal increase of heat, so that, on the whole, there was neither gain nor loss. In a second experiment Joule placed the two cylinders in two different vessels of water, and showed that the gain of heat in the one exactly balanced the loss in the other.

More exact experiments afterwards carried out by Joule and Thomson, and to which we have already had occasion to refer (see p. 169), have shown that in the free expansion of air, oxygen, and all permanent gases, except hydrogen, there is a slight cooling effect. A very small amount of energy is necessary to enable them to expand, even though they do no external work. The heat required under (*b*) (p. 177) is not absolutely zero, but is extremely small. With hydrogen, however, the reverse is the case. There is a slight heating effect, but it is very slight; heat is given out in the process of free expansion. Practically, then, these experiments show that almost the whole excess of the heat required to raise the temperature of a mass of air at constant pressure, over that required to produce the same rise at constant volume, is used in the work necessary to overcome the external pressure. This gives us a means of determining by experiment the value of the mechanical equivalent of heat; for, let v_0 be the volume of unit mass of air at pressure p_0, and let the coefficient of expansion by heat at constant pressure be a; then, if the temperature rise one degree the volume expands by $a v_0$; and the work done against the external pressure by this expansion is $p_0 a v_0$. Let c be the specific heat at constant pressure, c' that at constant volume; then the heat used in this expansion is $c - c'$, since we are working with unit of mass of gas; and the ratio of the work done to the heat supplied gives the value of J the mechanical equivalent of heat, *i.e.* the number of

units of work done by one unit of heat. Thus we find

$$J = \frac{a\, p_0\, v_0}{c - c'}.$$

If we remember that $a = \frac{1}{T_0}$, where T_0 is the temperature of the freezing point measured from absolute zero, we may write $J = \frac{p_0 v_0}{(c - c')\, T_0}$.

Now, if the pressure be that due to one atmosphere we have seen that $p_0 = 1{,}013{,}000$ dynes per square centimetre, while we have $v_0 = 773{\cdot}4$ cubic centimetres.

The values of c and c' respectively, as found by experiment, are ·2375 and ·1684, while $T_0 = 273$. On substituting these numbers we find $J = 41{\cdot}53 \times 10^6$ ergs.

This method of calculating J was first employed by Dr. Mayer, of Heilbronn, in 1842. He did not show, however, that he was justified in making the assumption that no heat was required to produce the free expansion of a gas, and Joule's experiments were necessary to complete the reasoning.

We have stated in several places that heat is used in changing the state or condition of a substance from solid to liquid, or liquid to gaseous, without change of temperature. In the language of the original discoverer of this fact such heat was said to be latent, or hidden, because it did not affect the temperature as indicated by the thermometer. In the solid state the

molecules of a substance are held together by the forces between them. When the state is changed to that of a liquid, work is done against these forces, and the energy to do this work arises from the heat supplied to produce the change. The heat energy is transformed into the additional potential energy possessed by the fluid particles, and this potential energy can be again obtained from the fluid by causing it to resume the solid form.

By the latent heat of fusion of a solid, then, is denoted the number of units of heat required to change a unit of mass of the solid to the liquid form without change of temperature, while the latent heat of evaporation of a liquid denotes the number of units of heat required to evaporate unit of mass of the liquid without change of temperature.

We have been dealing in the present chapter with a number of miscellaneous laws relating to matter, and chiefly, it is true, to properties of matter with respect to heat. We will close it with a law known as Avogadro's law, which has a most important place in the science of chemistry. We have reasons for believing that bodies are made up of a number of very minute parts called molecules, and that these molecules are continually in very rapid motion. According to Maxwell ('Encyclopædia Britannica,' Article 'Atom') 'a molecule is that minute portion of a substance which moves about as a whole, so that its parts, if it has any, do not part company during the motion of agitation of the gas.' It is the smallest mass into which we can conceive

the substance capable of division without a change in its chemical nature. It may be that a molecule is capable of division; but, if so, the parts into which it is divided have no longer the properties of the original substance, but of some other substances of which the original is composed. Thus, copper sulphate is a compound of copper, sulphur, and oxygen; a molecule of this substance contains all three in certain definite proportions. Such a molecule can be divided into the three, but ceases after division to be copper sulphate. A molecule of steam or water vapour contains hydrogen and oxygen, and we can divide water into these two components, but, when separated, they cease to be water. It may be that a molecule of a substance—hydrogen, for example—consists of two exactly equal and similar portions, and that in some changes the molecules are divided into these two portions; but, where this is the case, the separate parts of which the hydrogen molecule is composed have properties which differ from those of hydrogen, and are not hydrogen except when combined two and two.

A molecule of any substance has a certain definite mass; each molecule, if a compound, has a definite composition. Thus, in carbonic oxide, a molecule of carbon combines with one of oxygen to form the compound; hence there are as many molecules of oxygen present as of carbon, and the mass of oxygen which will combine with a given mass of carbon to form carbonic oxide is a perfectly definite quantity, and bears to the

mass of carbon the same ratio as the mass of a molecule of oxygen bears to that of a molecule of carbon.

By adding to each molecule of carbonic oxide a second molecule of oxygen we get carbonic acid; the mass of oxygen which this contains for the same mass of carbon will be twice as great as is contained in the carbonic oxide. According to this, then, we should expect the ratio of the masses in which two substances combine to form a third to be definite, and to have some simple numerical relation to the ratio of the masses of the combining molecules, and this is found to be the case.

Now, Avogadro's law states that equal volumes of all substances when in the state of gas, and under like conditions of pressure and temperature, contain the same number of molecules. It was enunciated by him in 1811, and again by Ampère in 1814.

Thus, suppose that at a temperature of 0° C., and a pressure due to 760 mm. of mercury, there are a certain definite number of molecules in a litre of hydrogen gas; there are exactly the same number of molecules in a litre of oxygen, nitrogen, carbonic oxide, or any other gas or vapour at the same pressure and temperature.

Again, suppose we find the density, that is, the mass, of a cubic centimetre of each of two gases at the same pressure and temperature; since the number of molecules in each of these two gases is the same, the ratio of the two masses found above is the ratio of the masses of the individual molecules of the two gases. Thus, for

example, it has been found that the mass of 1000 cc. of hydrogen at 0° C. and 760 is ·0896 gramme, while the mass of 1000 cc. of oxygen under the same conditions is just sixteen times this quantity. Thus the mass of a molecule of oxygen gas is sixteen times that of a molecule of hydrogen gas.

If, then, we take the mass of a molecule of any given substance as unity, we can attach to any other substance a definite number which represents on this system the mass of a molecule of that substance. It really of course gives the ratio of the mass of that molecule to the standard molecule. These numbers are known as the molecular weights of the substances.

As a rule, the mass of a hydrogen molecule is taken as unity and other substances are referred to it.

We obtain thus a list such as the following, which gives the names of some of the chemical elements and their molecular weights:

Carbon . . .	12	Lithium . . .	7
Copper . . .	63·3	Mercury . . .	200
Gold	197	Nitrogen . . .	14
Hydrogen . . .	1	Oxygen . . .	16
Iron	56	Zinc	65·2

To discuss more fully the laws which regulate the combination of molecules to form different substances is the province of Chemistry. In the present volume we have been dealing rather with the mechanical and physical properties of bodies than with the nature of the substances of which they are composed.

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